

Response surface regressions for critical value bounds and approximate p-values in equilibrium correction models

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Appendix B Details on the computational methods

In the following, we present some computational aspects about the Monte Carlo simulations in Section 3 of the main paper. All computations are performed in *Stata* 15. The bulk of the computations, the Monte Carlo simulations, are performed in *Stata*'s integrated matrix language, *Mata*. As a byte-compiled language, *Mata* runs about 5 to 6 times slower than a high-performance, compiled language such as *C*. However, most *Mata* functions used in our simulations hook directly into compiled ones, such as *LAPACK* functions (Anderson et al., 1999), which decreases the speed disadvantage substantially. We estimate that our simulation runs about half as fast as pure *C* would. *Mata*, however, is much more user friendly than *C*. For example, an appropriate random number generation mechanism that has a sufficiently large period and that accommodates parallel computations is readily available. For that, we use random number streams based on the Mersenne Twister pseudorandom number generator. Overall, we believe that *Mata* provides a good balance between speed and high-level language features. We run our computations in parallel on 35 cores, each of which running at 2.9 GHz. After the removal of any redundant calculations, such as repeated calculation of the same cross products, the simulations conclude after about three days.

Storing the calculated statistics is a desirable computational aspect of the simulation.

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One of the advantages is that it isolates sequential steps that are computationally intensive. Once the statistics are saved, any subsequent operations can be done independently, without re-calculating the results from the previous step over and over again, should either bugs or additional research ideas pop up. However, the large number of calculated statistics, roughly 100 billion F -statistics and 60 billion t -statistics, poses several problems, the most serious one being storage. Using floating point numbers with 8 digit precision (4 bytes per number), the (uncompressed) storage requirement is 640 GB. While this is not technically infeasible, it is too much of a hindrance for practical research. Our solution was to round the calculated statistics to three digits after the decimal point. It is important to note that the effect of rounding on the RS regressions is absolutely negligible. We then further transformed the rounded numbers in terms of first differences of sorted statistics and occurrence counts. The transformation is completely reversible, so that the original rounded 10 billion statistics per simulation design can be fully recovered. The resulting storage requirements are 40 GB, which decrease further to 8 GB when adding a conventional compression algorithm. This magnitude is easily manageable.

Appendix C Critical values and approximate p -values

To assess the precision of the empirical distribution functions obtained in our Monte Carlo simulation in Section 3.1 of the main paper, we can compute the coefficient of variation for the quantiles of interest based on the 100 meta replications with 100,000 replications each. For selected simulation designs, they are reported in Tables 5 and 6. Because the replications for a given design are independent, the coefficient of variation for the quantiles based on 10 million replications is expected to be one-tenth of the one for 100,000 replications.

Besides being useful on their own in an empirical analysis, the approximate p -values computed in Section 3.3 of the main paper can be used to assess the relevance of the differences between asymptotic and finite-sample critical values. Tables 7 and 8 present the approximate finite-sample p -values for a given sample size and variable count that correspond to the respective asymptotic critical values at a specified significance level. These p -values can be interpreted as the expected finite-sample size of the asymptotic test.

Appendix D Separate response surface regressions

In Section 3.2 of the main paper, we have obtained RS estimates for the F - and t -statistic that allow us to predict quantiles of the distributions for any number of long-run forcing variables. In the previous literature, these RS models were estimated separately for each variable count k of interest. In this appendix, we do the same for all $k \in [0, 10]$. While the resulting predictions are expected to be slightly more precise, we have seen in Figure 3 of the main paper that there is hardly any practically relevant difference compared to the joint model for all k .

In the following, we estimate separate RS models for each quadruplet $\{c, k, d, p\}$. Given the 100 meta replications, up to 19 choices of the time horizon T , and 8 different lag orders q , we have between 5,900 and 12,400 observations per estimation, accounting for the restriction that there shall be at least twice as many observations as parameters in equation (6).¹ The RS model is

$$Q_k(T, q) = \sum_{j=0}^m \sum_{l=0}^n \theta_{j,l} [N(T, q)]^{-j} [H(q, k)]^l + u, \quad (\text{D.1})$$

where $Q_k(T, q)$ is the respective quantile from each meta replication for a given k , $N(T, q)$ is the effective sample size, and $H(q, k)$ the number of unrestricted short-run coefficients. The presence of stationary first-differenced terms in equation (6) when $q > 0$ does not affect the asymptotic properties of the distribution which implies the restrictions $\theta_{0,l} = 0$ for all $l > 0$. The intercept $\theta_{0,0}$ can then be interpreted as the asymptotic quantile when $T \rightarrow \infty$. We have chosen the polynomial orders $m = 3$ and $n = 1$. The latter provides a better fit than alternatively setting $n = 3$ together with the restrictions $\theta_{j,l} = 0$ whenever $j \neq l$ for $l > 0$, which has been done by Cheung and Lai (1995). Equation (D.1) thus reduces to

$$Q_k(T, q) = \theta_{0,0} + \sum_{j=1}^3 \theta_{j,0} \frac{1}{[N(T, q)]^j} + \sum_{j=1}^3 \theta_{j,1} \frac{H(q, k)}{[N(T, q)]^j} + u. \quad (\text{D.2})$$

In Appendix E, we report the ordinary least squares results for the quantiles corresponding to a size of 1%, 5%, and 10%.² Tables 9 to 16 also contain the standard error

¹The largest number of observations is available for $k = 1$ in case (i), and the smallest number for $k = 10$ in cases (iv) and (v).

²Estimates for other quantiles are available upon request.

(SE) of the intercept, robust to heteroskedasticity, as a measure of uncertainty about the asymptotic quantile. It is always smaller than 0.0041 for the F -statistic and below 0.0011 for the t -statistic. In most experimental designs, the standard error remains far below this magnitude. However, the reported standard errors are too small because they are conditional on the correct specification of the RS model, as emphasized by MacKinnon (1991).

The asymptotic critical values can be read off directly from the RS intercept $\theta_{0,0}$. Our estimates are close to the corresponding near-asymptotic critical values tabulated by Pesaran et al. (2001). The absolute difference is for the most part below 0.05, both for the F -statistic and the t -statistic. However, these asymptotic critical values are less useful in small samples. For a given number of variables in the level relationship, finite-sample critical values can be computed from the regression coefficients for any combination of the effective sample size and number of short-run coefficients.

Previously reported critical values typically do not take the lag augmentation in equation (6) into account and might thus be inaccurate in many empirically relevant situations, in particular when the sample size is relatively small. Figure 4 in the main paper illustrates the variation across lag orders. For $k = 0$, there is obviously no distinction possible between $I(0)$ and $I(1)$ variables in the level relationship. In this situation, the F -statistic in cases (ii) and (iv) is the one analyzed by Dickey and Fuller (1981). In cases (i), (iii), and (v), it equals the square of the t -statistic. The latter corresponds to the familiar augmented Dickey-Fuller unit-root test statistic. The asymptotic critical values obtained from our RS regressions closely match those reported in the previous literature.³

RS estimates for the original Dickey and Fuller (1979) test statistic, $q = 1$, have been previously obtained by MacKinnon (1991, 2010) and Ericsson and MacKinnon (2002).⁴ Cheung and Lai (1995) go one step further by estimating a RS that allows the quantiles of the distribution to vary with the lag order. Figure 7 compares these RS estimates to ours for case (iii) and three different lag orders at a size of 5%. For the test without lag augmentation, $q = 1$, our RS and the ones from MacKinnon (2010) and Ericsson and MacKinnon (2002) are visually indistinguishable and they all fit nicely through the

³See Table 1 in the main paper.

⁴Dickey (1976) obtains his critical values as predictions from RS regressions but he does not report the regression coefficients.

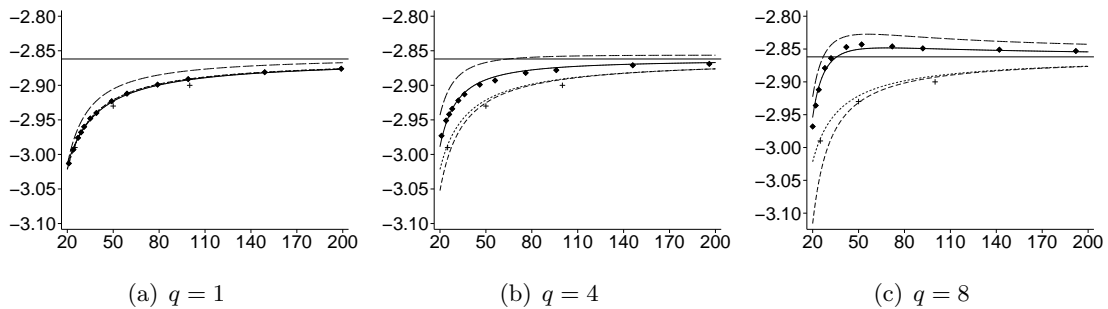


Figure 7: RS from equation (D.2) for the t -statistic in case (iii) with $k = 0$ variables at the 5% significance level for selected lag orders q over a range of effective sample sizes $N(T, q)$. The diamonds are the critical values computed from the aggregate EDFs of the 10^7 t -statistics. The horizontal line represents the respective estimate of $\theta_{0,0}$ in Table 15 and the solid curve the corresponding RS. The long-dashed curve is the RS from Cheung and Lai (1995), the medium-dashed curve from Ericsson and MacKinnon (2002), and the short-dashed curve from MacKinnon (2010). Crosses are tabulated critical values from Dickey (1976).

quantiles from the aggregate EDFs obtained in Section 3.1 of the main paper.⁵

The advantage of our approach becomes apparent when we move to higher lag orders. Because the RS from MacKinnon (2010) does not accommodate the lag augmentation, it becomes too conservative. In fact, for higher lag orders the asymptotic critical value would provide a better approximation for most sample sizes than the MacKinnon (2010) surface or the tabulated critical values from Dickey (1976). By contrast, Figure 7 confirms that our RS provides a very good fit to the critical values implied by our simulated aggregate EDFs. It also outperforms the RS from Cheung and Lai (1995) that is skewed towards zero, possibly due to the smaller number of replications in their simulation and a lower polynomial order in their RS regressions. Ericsson and MacKinnon (2002) indirectly account for the lag order by estimating the RS over the degrees-of-freedom adjusted sample size. However, Figure 7 clearly shows that this strategy is not appropriate for higher lag orders as the fit worsens even compared to MacKinnon (2010).

In the multivariable environment, the order of integration affects the distribution of the test statistic. Banerjee et al. (1998) and Ericsson and MacKinnon (2002) consider the t -statistic for cointegration testing under the assumption that all regressors are individually $I(1)$, the upper bound for the bounds test, but neither of them account for the lag

⁵MacKinnon (2010) is an updated version of MacKinnon (1991).

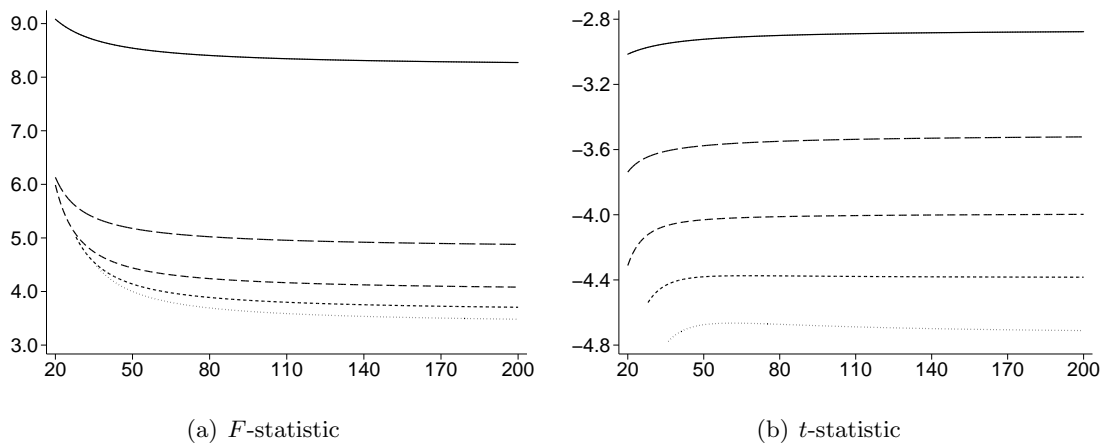


Figure 8: Upper-bound RS from equation (D.2) for the F - and t -statistic in case (iii) at the 5% significance level with $k \in \{0, 2, 4, 6, 8\}$ variables over a range of effective sample sizes $N(T, q)$ and with a lag order $q = 1$. The solid curve refers to $k = 0$. With increasing k , the curves have shorter dashes.

augmentation. In this situation, when we vary k for a fixed lag order $q = 1$, the spread between the RS curves is largely driven by the asymptotic critical value that now depends on k . This is shown in Figure 8 for both the F - and t -statistic. Importantly, the gap between the curves becomes systematically smaller with increasing k , which justifies our approach in Section 3.2 of the main paper to directly model the variation in k as part of a joint RS.

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Appendix E Tables

Table 5: Coefficient of variation, F -statistic

T	k	1%		5%		10%	
		$I(0)$	$I(1)$	$I(0)$	$I(1)$	$I(0)$	$I(1)$
Case (i)							
30	0	0.0082	0.0082	0.0065	0.0065	0.0059	0.0059
	2	0.0067	0.0066	0.0045	0.0042	0.0038	0.0034
	4	0.0064	0.0060	0.0039	0.0035	0.0032	0.0031
80	0	0.0083	0.0083	0.0070	0.0070	0.0060	0.0060
	2	0.0066	0.0060	0.0037	0.0036	0.0035	0.0030
	4	0.0056	0.0048	0.0033	0.0030	0.0030	0.0024
1000	8	0.0044	0.0043	0.0030	0.0027	0.0027	0.0021
	0	0.0075	0.0075	0.0062	0.0062	0.0051	0.0051
	2	0.0058	0.0050	0.0038	0.0029	0.0033	0.0027
	4	0.0053	0.0038	0.0035	0.0024	0.0030	0.0024
	8	0.0044	0.0030	0.0028	0.0020	0.0024	0.0015
	Case (ii)						
30	0	0.0065	0.0065	0.0038	0.0038	0.0029	0.0029
	2	0.0056	0.0058	0.0036	0.0033	0.0030	0.0027
	4	0.0064	0.0063	0.0036	0.0033	0.0028	0.0026
80	0	0.0054	0.0054	0.0036	0.0036	0.0030	0.0030
	2	0.0052	0.0051	0.0031	0.0030	0.0026	0.0027
	4	0.0046	0.0044	0.0027	0.0027	0.0023	0.0021
1000	8	0.0043	0.0038	0.0024	0.0024	0.0022	0.0019
	0	0.0055	0.0055	0.0034	0.0034	0.0029	0.0029
	2	0.0049	0.0045	0.0027	0.0023	0.0023	0.0022
	4	0.0041	0.0035	0.0025	0.0022	0.0022	0.0020
	8	0.0035	0.0030	0.0020	0.0020	0.0020	0.0015
	Case (iii)						
30	0	0.0069	0.0069	0.0042	0.0042	0.0033	0.0033
	2	0.0068	0.0057	0.0039	0.0035	0.0033	0.0027
	4	0.0060	0.0068	0.0036	0.0034	0.0029	0.0028
80	0	0.0061	0.0061	0.0042	0.0042	0.0035	0.0035
	2	0.0053	0.0054	0.0032	0.0032	0.0031	0.0030
	4	0.0047	0.0045	0.0028	0.0028	0.0028	0.0023
1000	8	0.0046	0.0040	0.0024	0.0026	0.0022	0.0020
	0	0.0056	0.0056	0.0035	0.0035	0.0030	0.0030
	2	0.0052	0.0047	0.0029	0.0029	0.0025	0.0026
	4	0.0042	0.0037	0.0026	0.0023	0.0024	0.0020
	8	0.0037	0.0031	0.0023	0.0020	0.0020	0.0016
	Case (iv)						
30	0	0.0061	0.0061	0.0034	0.0034	0.0027	0.0027
	2	0.0060	0.0056	0.0034	0.0032	0.0030	0.0025
	4	0.0054	0.0062	0.0034	0.0034	0.0026	0.0027
80	0	0.0052	0.0052	0.0035	0.0035	0.0028	0.0028
	2	0.0055	0.0045	0.0030	0.0031	0.0024	0.0022
	4	0.0042	0.0039	0.0026	0.0028	0.0023	0.0021
1000	8	0.0039	0.0036	0.0025	0.0021	0.0020	0.0017
	0	0.0045	0.0045	0.0027	0.0027	0.0024	0.0024
	2	0.0039	0.0043	0.0025	0.0022	0.0020	0.0020
	4	0.0040	0.0035	0.0025	0.0021	0.0021	0.0017
	8	0.0036	0.0028	0.0022	0.0018	0.0020	0.0014
	Case (v)						
30	0	0.0067	0.0067	0.0034	0.0034	0.0029	0.0029
	2	0.0061	0.0063	0.0037	0.0034	0.0030	0.0027
	4	0.0056	0.0066	0.0036	0.0036	0.0028	0.0027
80	0	0.0056	0.0056	0.0040	0.0040	0.0029	0.0029
	2	0.0056	0.0043	0.0029	0.0032	0.0026	0.0025
	4	0.0046	0.0039	0.0026	0.0029	0.0023	0.0023
1000	8	0.0041	0.0039	0.0027	0.0020	0.0021	0.0018
	0	0.0047	0.0047	0.0028	0.0028	0.0024	0.0024
	2	0.0043	0.0040	0.0026	0.0022	0.0021	0.0020
	4	0.0040	0.0035	0.0025	0.0022	0.0021	0.0018
	8	0.0038	0.0027	0.0022	0.0018	0.0020	0.0015

Note: The coefficient of variation is computed as the ratio of the standard deviation to the mean over the 100 meta replications for the empirical quantiles that correspond to the respective significance level and simulation design. Only designs with a lag order $q = 1$ are considered.

Table 6: Coefficient of variation, t -statistic

T	k	1%		5%		10%	
		$I(0)$	$I(1)$	$I(0)$	$I(1)$	$I(0)$	$I(1)$
Case (i)							
30	0	0.0046	0.0046	0.0040	0.0040	0.0031	0.0031
	2	0.0050	0.0036	0.0040	0.0023	0.0036	0.0020
	4	0.0048	0.0037	0.0043	0.0024	0.0036	0.0021
80	0	0.0044	0.0044	0.0040	0.0040	0.0032	0.0032
	2	0.0041	0.0034	0.0038	0.0020	0.0033	0.0018
	4	0.0048	0.0027	0.0035	0.0019	0.0035	0.0018
1000	8	0.0051	0.0031	0.0034	0.0016	0.0036	0.0015
	0	0.0042	0.0042	0.0031	0.0031	0.0032	0.0032
	2	0.0043	0.0029	0.0031	0.0021	0.0031	0.0019
	4	0.0044	0.0027	0.0031	0.0017	0.0031	0.0016
	8	0.0044	0.0023	0.0032	0.0013	0.0033	0.0013
	Case (iii)						
30	0	0.0035	0.0035	0.0021	0.0021	0.0017	0.0017
	2	0.0037	0.0036	0.0023	0.0020	0.0019	0.0015
	4	0.0042	0.0032	0.0025	0.0020	0.0021	0.0017
80	0	0.0031	0.0031	0.0021	0.0021	0.0018	0.0018
	2	0.0030	0.0030	0.0021	0.0019	0.0017	0.0017
	4	0.0033	0.0026	0.0020	0.0017	0.0019	0.0015
1000	8	0.0035	0.0027	0.0023	0.0016	0.0019	0.0013
	0	0.0028	0.0028	0.0017	0.0017	0.0015	0.0015
	2	0.0030	0.0025	0.0017	0.0016	0.0015	0.0015
	4	0.0029	0.0020	0.0018	0.0016	0.0014	0.0012
	8	0.0030	0.0019	0.0018	0.0012	0.0015	0.0011
	Case (v)						
30	0	0.0033	0.0033	0.0017	0.0017	0.0015	0.0015
	2	0.0034	0.0033	0.0023	0.0018	0.0016	0.0017
	4	0.0036	0.0035	0.0023	0.0020	0.0019	0.0017
80	0	0.0028	0.0028	0.0020	0.0020	0.0015	0.0015
	2	0.0026	0.0023	0.0020	0.0017	0.0015	0.0015
	4	0.0027	0.0024	0.0019	0.0015	0.0016	0.0014
1000	8	0.0030	0.0025	0.0019	0.0014	0.0017	0.0013
	0	0.0024	0.0024	0.0014	0.0014	0.0012	0.0012
	2	0.0023	0.0021	0.0014	0.0013	0.0012	0.0011
	4	0.0023	0.0019	0.0014	0.0013	0.0012	0.0012
	8	0.0026	0.0018	0.0014	0.0011	0.0012	0.0009

Note: The coefficient of variation is computed as the ratio of the standard deviation to the absolute value of the mean over the 100 meta replications for the empirical quantiles that correspond to the respective significance level and simulation design. Only designs with a lag order $q = 1$ are considered.

Table 7: Finite-sample p -values, F -statistic

T	k	1%		5%		10%	
		$I(0)$	$I(1)$	$I(0)$	$I(1)$	$I(0)$	$I(1)$
Case (i)							
30	0	0.0141	0.0131	0.0572	0.0554	0.1075	0.1054
	2	0.0219	0.0255	0.0713	0.0765	0.1235	0.1283
	4	0.0325	0.0423	0.0882	0.1018	0.1422	0.1555
80	0	0.0110	0.0102	0.0517	0.0501	0.1017	0.0997
	2	0.0125	0.0129	0.0550	0.0559	0.1058	0.1064
	4	0.0147	0.0168	0.0595	0.0639	0.1110	0.1159
1000	0	0.0210	0.0300	0.0700	0.0858	0.1219	0.1402
	2	0.0100	0.0100	0.0501	0.0499	0.1001	0.0999
	4	0.0101	0.0101	0.0502	0.0502	0.1003	0.1002
	8	0.0102	0.0103	0.0505	0.0507	0.1005	0.1009
	8	0.0105	0.0109	0.0510	0.0522	0.1011	0.1027
Case (ii)							
30	0	0.0187	0.0169	0.0661	0.0633	0.1177	0.1148
	2	0.0280	0.0341	0.0805	0.0919	0.1324	0.1465
	4	0.0402	0.0560	0.0974	0.1223	0.1495	0.1785
80	0	0.0120	0.0107	0.0540	0.0515	0.1044	0.1016
	2	0.0137	0.0145	0.0573	0.0597	0.1078	0.1115
	4	0.0162	0.0196	0.0617	0.0696	0.1124	0.1231
1000	0	0.0222	0.0340	0.0707	0.0931	0.1210	0.1490
	2	0.0101	0.0100	0.0502	0.0499	0.1002	0.0999
	4	0.0102	0.0101	0.0503	0.0504	0.1003	0.1005
	8	0.0103	0.0104	0.0506	0.0510	0.1006	0.1013
	8	0.0106	0.0111	0.0510	0.0526	0.1010	0.1032
Case (iii)							
30	0	0.0170	0.0161	0.0609	0.0594	0.1098	0.1083
	2	0.0243	0.0299	0.0728	0.0828	0.1224	0.1343
	4	0.0358	0.0495	0.0894	0.1107	0.1397	0.1638
80	0	0.0121	0.0113	0.0531	0.0519	0.1025	0.1012
	2	0.0132	0.0141	0.0553	0.0579	0.1048	0.1084
	4	0.0154	0.0185	0.0593	0.0664	0.1091	0.1183
1000	0	0.0214	0.0318	0.0685	0.0881	0.1184	0.1422
	2	0.0101	0.0101	0.0502	0.0500	0.1001	0.1000
	4	0.0102	0.0102	0.0503	0.0504	0.1002	0.1004
	8	0.0103	0.0104	0.0505	0.0509	0.1004	0.1010
	8	0.0106	0.0111	0.0510	0.0523	0.1009	0.1028
Case (iv)							
30	0	0.0237	0.0219	0.0753	0.0727	0.1285	0.1259
	2	0.0334	0.0409	0.0886	0.1019	0.1409	0.1573
	4	0.0468	0.0653	0.1055	0.1342	0.1567	0.1905
80	0	0.0135	0.0122	0.0571	0.0550	0.1082	0.1059
	2	0.0149	0.0161	0.0595	0.0629	0.1101	0.1153
	4	0.0174	0.0216	0.0633	0.0731	0.1137	0.1269
1000	0	0.0233	0.0361	0.0717	0.0962	0.1213	0.1524
	2	0.0102	0.0101	0.0504	0.0502	0.1005	0.1002
	4	0.0103	0.0103	0.0505	0.0506	0.1005	0.1008
	8	0.0104	0.0105	0.0507	0.0513	0.1006	0.1016
	8	0.0107	0.0112	0.0511	0.0527	0.1009	0.1034
Case (v)							
30	0	0.0214	0.0204	0.0686	0.0674	0.1186	0.1174
	2	0.0287	0.0354	0.0792	0.0909	0.1285	0.1427
	4	0.0415	0.0576	0.0958	0.1206	0.1445	0.1734
80	0	0.0135	0.0128	0.0559	0.0549	0.1058	0.1047
	2	0.0142	0.0154	0.0569	0.0604	0.1063	0.1112
	4	0.0162	0.0201	0.0602	0.0691	0.1094	0.1213
1000	0	0.0220	0.0335	0.0685	0.0904	0.1172	0.1448
	2	0.0103	0.0102	0.0504	0.0503	0.1004	0.1003
	4	0.0103	0.0103	0.0504	0.0505	0.1003	0.1006
	8	0.0104	0.0105	0.0505	0.0511	0.1003	0.1012
	8	0.0107	0.0112	0.0509	0.0524	0.1006	0.1029

Note: Reported are the approximate finite-sample p -values obtained from equation (10) that are associated with the asymptotic critical value for a given significance level. Only designs with a lag order $q = 1$ are considered.

Table 8: Finite-sample p -values, t -statistic

T	k	1%		5%		10%	
		$I(0)$	$I(1)$	$I(0)$	$I(1)$	$I(0)$	$I(1)$
Case (i)							
30	0	0.0131	0.0116	0.0533	0.0515	0.1003	0.0990
	2	0.0143	0.0169	0.0543	0.0563	0.0995	0.0997
	4	0.0155	0.0203	0.0550	0.0578	0.0986	0.0967
80	0	0.0107	0.0099	0.0508	0.0505	0.1000	0.1009
	2	0.0110	0.0116	0.0510	0.0516	0.0994	0.0998
	4	0.0113	0.0126	0.0510	0.0515	0.0988	0.0972
1000	0	0.0119	0.0121	0.0512	0.0438	0.0976	0.0802
	2	0.0100	0.0100	0.0500	0.0501	0.1000	0.1002
	4	0.0100	0.0101	0.0500	0.0501	0.0999	0.1000
	8	0.0101	0.0101	0.0500	0.0500	0.0999	0.0997
	8	0.0101	0.0101	0.0500	0.0492	0.0998	0.0978
Case (iii)							
30	0	0.0169	0.0153	0.0612	0.0594	0.1106	0.1093
	2	0.0177	0.0200	0.0578	0.0606	0.1009	0.1029
	4	0.0179	0.0234	0.0542	0.0611	0.0920	0.0984
80	0	0.0117	0.0111	0.0531	0.0536	0.1030	0.1055
	2	0.0118	0.0125	0.0516	0.0532	0.0990	0.1012
	4	0.0118	0.0135	0.0498	0.0527	0.0947	0.0976
1000	0	0.0114	0.0125	0.0458	0.0435	0.0858	0.0786
	2	0.0101	0.0100	0.0502	0.0503	0.1002	0.1006
	4	0.0101	0.0101	0.0501	0.0502	0.0999	0.1001
	8	0.0101	0.0102	0.0499	0.0501	0.0995	0.0997
	8	0.0100	0.0101	0.0495	0.0491	0.0986	0.0976
Case (v)							
30	0	0.0208	0.0189	0.0683	0.0666	0.1188	0.1178
	2	0.0208	0.0235	0.0616	0.0654	0.1032	0.1071
	4	0.0205	0.0265	0.0557	0.0644	0.0905	0.1002
80	0	0.0126	0.0121	0.0553	0.0565	0.1060	0.1094
	2	0.0125	0.0135	0.0525	0.0551	0.0993	0.1029
	4	0.0123	0.0144	0.0496	0.0537	0.0929	0.0979
1000	0	0.0112	0.0128	0.0432	0.0434	0.0797	0.0773
	2	0.0101	0.0101	0.0503	0.0505	0.1004	0.1009
	4	0.0101	0.0102	0.0501	0.0504	0.0998	0.1003
	8	0.0101	0.0103	0.0498	0.0502	0.0993	0.0997
	8	0.0100	0.0101	0.0492	0.0490	0.0979	0.0973

Note: Reported are the approximate finite-sample p -values obtained from equation (10) that are associated with the asymptotic critical value for a given significance level. Only designs with a lag order $q = 1$ are considered.

Table 9: Response surface estimates, F -statistic, case (i)

k	α	$\theta_{0,0}$	$\theta_{1,0}$	$\theta_{2,0}$	$\theta_{3,0}$	$\theta_{1,1}$	$\theta_{2,1}$	$\theta_{3,1}$	SE($\theta_{0,0}$)	\bar{R}^2	RMSE
I(0)											
0	1%	6.8875	29.187	-48.36	706.4	-0.750	-0.47	813.1	0.0017	0.986	0.091
	5%	4.1053	12.271	-94.58	958.5	-0.627	23.14	-31.7	0.0006	0.983	0.033
	10%	2.9626	7.471	-91.34	862.4	-0.431	20.85	-111.8	0.0004	0.978	0.021
1	1%	4.7135	22.668	34.02	839.3	-0.302	-11.11	1135.8	0.0011	0.990	0.062
	5%	3.1042	9.650	-0.94	317.0	-0.288	1.26	313.1	0.0004	0.988	0.025
	10%	2.4078	5.744	-9.46	226.0	-0.237	2.52	144.3	0.0003	0.983	0.017
2	1%	3.8491	21.822	-47.64	3414.6	-0.619	13.15	789.1	0.0009	0.994	0.047
	5%	2.6738	9.348	-4.25	1092.2	-0.470	11.95	142.4	0.0004	0.992	0.021
	10%	2.1503	5.641	-0.33	577.7	-0.382	8.95	34.5	0.0002	0.988	0.014
3	1%	3.3558	23.794	-132.74	5112.0	-0.491	-10.91	1419.0	0.0007	0.996	0.035
	5%	2.4140	11.339	-62.13	1957.7	-0.400	-4.75	581.3	0.0004	0.994	0.018
	10%	1.9873	7.284	-44.03	1176.8	-0.341	-3.53	362.5	0.0002	0.991	0.013
4	1%	3.0386	24.475	-149.53	5806.6	-0.395	-34.16	2302.0	0.0007	0.997	0.030
	5%	2.2442	11.792	-51.95	1943.6	-0.377	-15.14	1010.2	0.0004	0.995	0.016
	10%	1.8787	7.765	-35.93	1088.3	-0.338	-10.74	664.7	0.0003	0.992	0.013
5	1%	2.8243	23.079	-101.35	6237.5	-0.519	-24.09	2469.0	0.0006	0.997	0.025
	5%	2.1241	12.010	-62.03	2502.8	-0.430	-13.81	1247.4	0.0003	0.996	0.015
	10%	1.7999	8.228	-54.36	1575.3	-0.377	-11.00	881.2	0.0003	0.993	0.012
6	1%	2.6584	23.357	-163.53	8573.9	-0.566	-19.04	2819.3	0.0006	0.998	0.021
	5%	2.0323	12.195	-85.76	3484.8	-0.473	-9.34	1436.7	0.0003	0.997	0.012
	10%	1.7397	8.387	-69.49	2189.8	-0.418	-6.99	1015.9	0.0002	0.996	0.010
7	1%	2.5221	25.664	-321.24	12416.1	-0.538	-27.69	3450.4	0.0007	0.998	0.022
	5%	1.9556	13.404	-150.84	5045.0	-0.459	-14.47	1814.9	0.0004	0.997	0.013
	10%	1.6885	9.300	-114.41	3206.4	-0.410	-11.00	1310.9	0.0003	0.996	0.010
8	1%	2.4126	28.117	-527.16	17583.6	-0.563	-27.01	3942.8	0.0006	0.999	0.021
	5%	1.8947	14.143	-209.19	6797.3	-0.478	-13.14	2097.6	0.0003	0.998	0.012
	10%	1.6481	9.658	-141.47	4161.9	-0.428	-9.50	1516.9	0.0003	0.997	0.010
9	1%	2.3251	28.065	-556.19	20221.0	-0.550	-28.71	4369.8	0.0007	0.998	0.020
	5%	1.8434	14.658	-244.19	8265.1	-0.473	-14.78	2391.1	0.0004	0.997	0.012
	10%	1.6130	10.262	-177.01	5271.3	-0.426	-11.19	1768.8	0.0003	0.996	0.010
10	1%	2.2538	27.294	-558.99	22795.4	-0.556	-30.81	5322.2	0.0006	0.998	0.017
	5%	1.8014	14.493	-256.90	9683.7	-0.474	-18.00	3180.1	0.0003	0.998	0.011
	10%	1.5848	10.022	-178.29	6106.3	-0.429	-13.61	2425.4	0.0003	0.996	0.009
I(1)											
0	1%	6.8875	29.187	-48.36	706.4	-0.750	-0.47	813.1	0.0017	0.986	0.091
	5%	4.1053	12.271	-94.58	958.5	-0.627	23.14	-31.7	0.0006	0.983	0.033
	10%	2.9626	7.471	-91.34	862.4	-0.431	20.85	-111.8	0.0004	0.978	0.021
1	1%	5.8446	27.970	39.91	878.2	-0.073	-5.16	1518.2	0.0012	0.993	0.069
	5%	4.0493	11.417	5.45	320.8	-0.163	7.21	390.9	0.0005	0.993	0.025
	10%	3.2454	6.503	-7.83	272.0	-0.128	6.45	171.3	0.0003	0.992	0.016
2	1%	5.1368	31.304	-117.99	4609.6	0.008	12.82	1609.5	0.0012	0.995	0.061
	5%	3.7851	13.214	-24.12	1403.0	-0.086	14.90	417.1	0.0005	0.996	0.023
	10%	3.1598	7.680	-11.09	726.4	-0.075	10.86	186.3	0.0003	0.995	0.014
3	1%	4.7040	29.674	-6.19	5284.2	-0.063	36.43	1218.7	0.0008	0.997	0.043
	5%	3.5887	13.409	25.28	1560.5	-0.061	21.23	348.3	0.0004	0.997	0.017
	10%	3.0652	8.165	16.41	827.4	-0.049	14.00	166.4	0.0002	0.997	0.011
4	1%	4.3928	31.836	-64.25	7647.1	0.083	18.61	1967.8	0.0010	0.998	0.039
	5%	3.4371	14.924	7.57	2437.9	0.007	14.90	651.6	0.0004	0.998	0.016
	10%	2.9832	9.189	11.98	1278.7	-0.006	10.75	339.8	0.0003	0.998	0.010
5	1%	4.1779	29.702	86.47	7701.6	-0.045	41.15	1616.5	0.0008	0.998	0.032
	5%	3.3265	14.408	81.75	2306.6	-0.041	24.65	524.6	0.0003	0.998	0.014
	10%	2.9192	9.106	55.87	1216.7	-0.030	16.36	281.1	0.0002	0.998	0.009
6	1%	3.9941	32.939	-33.92	11544.7	0.062	24.65	2267.8	0.0007	0.999	0.028
	5%	3.2303	16.095	42.64	3757.3	0.015	17.37	821.6	0.0003	0.999	0.012
	10%	2.8616	10.233	38.51	1997.2	0.003	12.49	450.9	0.0002	0.999	0.008
7	1%	3.8503	35.919	-212.97	17139.0	0.042	26.35	2565.3	0.0008	0.999	0.027
	5%	3.1535	17.427	-3.04	5639.3	0.009	18.55	942.1	0.0003	0.999	0.011
	10%	2.8143	11.150	15.73	3065.6	0.001	13.38	528.9	0.0002	0.999	0.007
8	1%	3.7253	41.566	-573.99	26110.7	0.045	20.27	3137.6	0.0009	0.999	0.029
	5%	3.0868	19.641	-113.62	8761.4	0.010	17.45	1157.4	0.0003	0.999	0.011
	10%	2.7728	12.491	-39.84	4781.1	0.001	13.15	650.2	0.0002	0.999	0.007
9	1%	3.6300	41.281	-562.94	29869.2	0.061	19.85	3545.5	0.0009	0.999	0.027
	5%	3.0327	19.932	-108.85	10359.2	0.019	17.32	1364.9	0.0004	0.999	0.011
	10%	2.7379	12.842	-36.82	5757.2	0.009	12.79	795.7	0.0002	0.999	0.007
10	1%	3.5472	42.192	-587.36	33958.3	0.065	18.73	3494.3	0.0008	0.999	0.022
	5%	2.9848	21.090	-146.81	12623.0	0.021	16.97	1320.1	0.0003	0.999	0.009
	10%	2.7069	13.660	-56.81	7099.3	0.011	12.79	762.8	0.0002	0.999	0.006

Note: The RS regression model is equation (D.2). The dependent variable is the simulated α -quantile of the test statistic. Separate regressions are run for each number k of individually $I(0)$ or $I(1)$ variables \mathbf{x}_t in equation (6). SE($\theta_{0,0}$) denotes the heteroskedasticity-robust standard error of the intercept, \bar{R}^2 the adjusted coefficient of determination, and RMSE the root mean square error.

Table 10: Response surface estimates, F -statistic, case (ii)

k	α	$\theta_{0,0}$	$\theta_{1,0}$	$\theta_{2,0}$	$\theta_{3,0}$	$\theta_{1,1}$	$\theta_{2,1}$	$\theta_{3,1}$	$SE(\theta_{0,0})$	\bar{R}^2	RMSE
I(0)											
0	1%	6.3769	28.932	226.25	-1520.9	-0.906	-60.16	2021.6	0.0012	0.993	0.075
	5%	4.5831	12.625	79.12	-667.0	-1.117	-9.37	574.0	0.0005	0.990	0.032
	10%	3.7792	7.444	42.51	-408.1	-0.987	-2.23	284.6	0.0003	0.984	0.021
1	1%	4.8785	26.267	69.67	2336.8	-1.172	9.45	1078.3	0.0009	0.995	0.050
	5%	3.5974	11.744	38.59	662.4	-0.989	11.09	279.8	0.0004	0.995	0.020
	10%	3.0150	7.141	19.81	387.0	-0.869	9.11	114.0	0.0002	0.994	0.013
2	1%	4.0934	26.566	-31.44	4698.6	-0.956	-3.40	1452.7	0.0008	0.997	0.040
	5%	3.0836	12.090	10.41	1514.4	-0.863	5.80	425.2	0.0003	0.997	0.016
	10%	2.6175	7.422	12.95	778.1	-0.767	4.90	214.2	0.0002	0.996	0.011
3	1%	3.6031	28.947	-198.99	7953.9	-0.816	-19.97	2017.6	0.0008	0.998	0.033
	5%	2.7620	13.307	-38.61	2488.8	-0.748	-5.75	782.8	0.0003	0.998	0.014
	10%	2.3688	8.408	-14.23	1282.1	-0.678	-3.86	480.8	0.0002	0.997	0.010
4	1%	3.2778	26.433	-85.00	7477.1	-0.738	-25.03	2407.6	0.0007	0.998	0.026
	5%	2.5448	12.919	-6.99	2460.0	-0.690	-9.88	1047.0	0.0003	0.998	0.013
	10%	2.2001	8.479	-1.63	1320.5	-0.635	-7.08	684.2	0.0002	0.997	0.009
5	1%	3.0379	27.168	-165.03	9944.4	-0.759	-29.36	2963.4	0.0006	0.999	0.022
	5%	2.3851	13.620	-47.51	3562.1	-0.694	-11.30	1338.2	0.0003	0.999	0.011
	10%	2.0766	9.082	-32.18	2074.7	-0.635	-7.74	898.4	0.0002	0.998	0.009
6	1%	2.8515	28.119	-284.10	13560.4	-0.774	-22.92	3199.6	0.0006	0.999	0.021
	5%	2.2611	14.333	-100.91	5084.5	-0.677	-11.30	1598.1	0.0003	0.999	0.010
	10%	1.9806	9.670	-68.75	3054.5	-0.618	-8.54	1124.7	0.0002	0.998	0.008
7	1%	2.6974	32.043	-578.21	20047.3	-0.743	-35.30	4044.9	0.0007	0.999	0.022
	5%	2.1606	16.000	-208.71	7526.5	-0.649	-18.00	2071.9	0.0003	0.999	0.011
	10%	1.9028	10.861	-137.52	4553.9	-0.591	-14.14	1508.2	0.0002	0.998	0.009
8	1%	2.5798	31.021	-580.72	22624.3	-0.735	-29.14	4298.2	0.0006	0.999	0.019
	5%	2.0812	15.912	-223.09	8837.0	-0.641	-15.72	2320.5	0.0003	0.999	0.010
	10%	1.8412	10.958	-152.31	5459.2	-0.585	-12.87	1731.7	0.0002	0.998	0.008
9	1%	2.4826	29.683	-551.60	24821.1	-0.722	-27.61	4987.9	0.0006	0.999	0.017
	5%	2.0159	15.438	-219.10	10044.9	-0.632	-15.91	2927.5	0.0003	0.999	0.009
	10%	1.7905	10.708	-154.09	6335.3	-0.576	-13.61	2284.5	0.0002	0.998	0.007
10	1%	2.3987	29.370	-543.71	26930.0	-0.702	-30.75	5412.1	0.0006	0.999	0.016
	5%	1.9586	15.912	-249.88	11602.6	-0.617	-17.75	3179.3	0.0003	0.998	0.010
	10%	1.7458	11.254	-188.26	7592.3	-0.565	-14.73	2458.4	0.0003	0.998	0.008
I(1)											
0	1%	6.3769	28.932	226.25	-1520.9	-0.906	-60.16	2021.6	0.0012	0.993	0.075
	5%	4.5831	12.625	79.12	-667.0	-1.117	-9.37	574.0	0.0005	0.990	0.032
	10%	3.7792	7.444	42.51	-408.1	-0.987	-2.23	284.6	0.0003	0.984	0.021
1	1%	5.4618	32.320	43.15	2824.7	-0.345	10.61	1590.0	0.0009	0.997	0.053
	5%	4.1084	15.078	19.97	915.9	-0.369	14.37	450.7	0.0004	0.997	0.020
	10%	3.4855	9.450	6.20	536.4	-0.324	11.02	215.4	0.0002	0.997	0.013
2	1%	4.9199	34.587	-40.57	5283.9	0.073	-1.69	2360.2	0.0011	0.997	0.052
	5%	3.8155	16.397	11.69	1555.0	-0.088	9.06	788.2	0.0004	0.998	0.020
	10%	3.2969	10.430	8.63	808.8	-0.101	7.50	431.8	0.0003	0.998	0.012
3	1%	4.5632	37.496	-222.89	10249.9	-0.047	25.14	2073.5	0.0010	0.998	0.042
	5%	3.6167	17.470	-20.18	3027.9	-0.086	20.55	665.5	0.0004	0.999	0.016
	10%	3.1663	11.076	2.00	1538.3	-0.085	15.45	342.9	0.0002	0.999	0.010
4	1%	4.3109	35.073	-60.22	10305.1	0.061	23.68	2355.8	0.0009	0.998	0.037
	5%	3.4712	17.189	37.78	3212.4	-0.029	21.23	773.1	0.0004	0.999	0.015
	10%	3.0679	11.196	33.82	1685.9	-0.040	15.86	414.8	0.0002	0.998	0.010
5	1%	4.1121	37.352	-142.31	13805.1	0.089	12.55	2922.8	0.0008	0.999	0.031
	5%	3.3551	18.606	10.03	4568.0	0.005	14.96	1063.2	0.0003	0.999	0.013
	10%	2.9892	12.159	22.13	2444.9	-0.014	12.46	582.7	0.0002	0.999	0.008
6	1%	3.9571	38.172	-215.56	18262.4	0.004	35.42	2538.4	0.0008	0.999	0.029
	5%	3.2641	18.945	9.01	5997.1	-0.022	25.23	911.7	0.0003	0.999	0.012
	10%	2.9266	12.474	26.64	3249.7	-0.026	18.71	492.5	0.0002	0.999	0.008
7	1%	3.8218	44.015	-604.70	27864.0	-0.019	29.05	3169.8	0.0010	0.999	0.031
	5%	3.1856	21.334	-117.79	9467.2	-0.033	23.84	1144.8	0.0004	0.999	0.012
	10%	2.8730	13.876	-35.90	5127.8	-0.030	18.20	624.9	0.0002	0.999	0.008
8	1%	3.7172	43.106	-546.56	31034.3	-0.008	37.79	2981.2	0.0008	0.999	0.027
	5%	3.1216	21.499	-105.36	11043.5	-0.022	27.46	1091.5	0.0003	0.999	0.011
	10%	2.8282	14.157	-28.12	6078.9	-0.022	20.33	609.9	0.0002	0.999	0.007
9	1%	3.6248	43.962	-570.88	35329.7	0.031	35.17	2827.7	0.0008	0.999	0.023
	5%	3.0651	22.375	-123.66	13046.4	-0.001	26.51	1004.8	0.0003	0.999	0.009
	10%	2.7884	14.939	-44.40	7392.6	-0.007	19.54	561.9	0.0002	0.999	0.006
10	1%	3.5518	41.779	-364.97	35425.4	0.040	31.98	3421.2	0.0008	0.999	0.019
	5%	3.0187	22.194	-68.96	13851.5	0.008	23.85	1357.8	0.0003	0.999	0.008
	10%	2.7554	15.028	-17.25	8047.5	0.000	17.86	795.0	0.0002	0.999	0.006

Note: The RS regression model is equation (D.2). The dependent variable is the simulated α -quantile of the test statistic. Separate regressions are run for each number k of individually $I(0)$ or $I(1)$ variables \mathbf{x}_t in equation (6). $SE(\theta_{0,0})$ denotes the heteroskedasticity-robust standard error of the intercept, \bar{R}^2 the adjusted coefficient of determination, and RMSE the root mean square error.

Table 11: Response surface estimates, F -statistic, case (iii)

k	α	$\theta_{0,0}$	$\theta_{1,0}$	$\theta_{2,0}$	$\theta_{3,0}$	$\theta_{1,1}$	$\theta_{2,1}$	$\theta_{3,1}$	$SE(\theta_{0,0})$	\bar{R}^2	RMSE
I(0)											
0	1%	11.7570	43.861	306.37	-2861.6	-4.037	-28.71	2832.9	0.0024	0.981	0.179
	5%	8.1893	16.491	77.09	-997.0	-3.686	27.36	580.0	0.0011	0.946	0.091
	10%	6.5903	8.444	21.87	-414.0	-3.113	26.22	182.6	0.0008	0.893	0.064
1	1%	6.8187	33.223	-28.85	4086.1	-2.015	42.84	993.4	0.0012	0.993	0.071
	5%	4.9055	13.345	-14.20	1463.9	-1.586	31.21	115.3	0.0005	0.989	0.033
	10%	4.0346	7.442	-24.28	997.6	-1.356	23.73	-31.1	0.0004	0.980	0.023
2	1%	5.1280	29.192	-16.16	4569.6	-1.136	-2.71	1783.4	0.0012	0.995	0.053
	5%	3.7841	12.223	17.43	1344.5	-1.009	7.63	521.5	0.0005	0.994	0.022
	10%	3.1638	6.808	22.06	578.7	-0.876	5.98	266.2	0.0003	0.992	0.015
3	1%	4.2658	29.088	-145.33	7753.9	-1.030	0.92	1706.0	0.0009	0.997	0.037
	5%	3.2112	12.389	-9.60	2302.1	-0.879	7.13	537.1	0.0003	0.997	0.015
	10%	2.7190	7.261	6.75	1141.8	-0.770	5.96	272.7	0.0002	0.997	0.010
4	1%	3.7410	26.457	-53.15	7531.7	-0.865	-8.31	2081.1	0.0007	0.998	0.029
	5%	2.8601	12.012	18.12	2328.2	-0.760	0.60	789.7	0.0003	0.998	0.012
	10%	2.4460	7.374	20.32	1177.7	-0.679	1.21	458.4	0.0002	0.998	0.008
5	1%	3.3828	27.150	-138.63	10004.3	-0.826	-17.91	2652.7	0.0006	0.998	0.024
	5%	2.6202	12.815	-22.08	3380.4	-0.720	-4.22	1091.7	0.0003	0.999	0.010
	10%	2.2599	8.123	-9.45	1878.0	-0.644	-2.10	683.5	0.0002	0.998	0.007
6	1%	3.1213	27.473	-228.84	13186.1	-0.818	-11.21	2791.9	0.0006	0.999	0.022
	5%	2.4453	13.348	-64.85	4740.2	-0.694	-3.27	1274.8	0.0003	0.999	0.009
	10%	2.1240	8.645	-38.20	2733.0	-0.619	-2.31	851.4	0.0002	0.999	0.007
7	1%	2.9146	31.561	-540.36	19890.5	-0.778	-24.28	3625.6	0.0007	0.999	0.022
	5%	2.3094	15.034	-172.63	7159.2	-0.660	-10.09	1724.9	0.0003	0.999	0.010
	10%	2.0189	9.801	-103.30	4172.4	-0.588	-7.54	1198.4	0.0002	0.999	0.007
8	1%	2.7598	30.262	-526.09	22151.7	-0.760	-17.91	3778.4	0.0006	0.999	0.019
	5%	2.2043	15.008	-187.17	8435.3	-0.642	-8.63	1946.6	0.0003	0.999	0.009
	10%	1.9372	9.981	-118.54	5049.5	-0.576	-6.84	1394.1	0.0002	0.999	0.007
9	1%	2.6331	29.426	-528.97	24762.7	-0.732	-19.06	4428.0	0.0006	0.999	0.017
	5%	2.1194	14.693	-186.17	9598.8	-0.626	-9.52	2451.1	0.0003	0.999	0.008
	10%	1.8712	9.926	-125.21	5929.0	-0.563	-8.12	1856.4	0.0002	0.999	0.006
10	1%	2.5287	28.641	-487.15	26207.1	-0.710	-22.09	4869.5	0.0006	0.999	0.015
	5%	2.0475	15.161	-215.58	11107.1	-0.608	-12.01	2746.5	0.0003	0.999	0.009
	10%	1.8152	10.448	-155.76	7105.5	-0.550	-9.62	2060.5	0.0002	0.998	0.007
I(1)											
0	1%	11.7570	43.861	306.37	-2861.6	-4.037	-28.71	2832.9	0.0024	0.981	0.179
	5%	8.1893	16.491	77.09	-997.0	-3.686	27.36	580.0	0.0011	0.946	0.091
	10%	6.5903	8.444	21.87	-414.0	-3.113	26.22	182.6	0.0008	0.893	0.064
1	1%	7.7358	41.914	-47.35	4635.9	-0.976	41.28	1598.1	0.0013	0.996	0.076
	5%	5.7040	18.262	-42.30	1849.9	-0.862	31.91	316.5	0.0006	0.994	0.035
	10%	4.7675	10.770	-48.49	1270.0	-0.755	23.10	88.0	0.0004	0.988	0.026
2	1%	6.2655	40.712	-65.42	5780.3	0.003	-4.17	2856.5	0.0014	0.997	0.065
	5%	4.7894	18.205	-1.16	1604.2	-0.174	6.49	984.5	0.0006	0.997	0.025
	10%	4.0949	10.958	-2.87	807.4	-0.194	4.77	556.8	0.0004	0.996	0.017
3	1%	5.4927	41.005	-219.01	10665.3	-0.153	35.87	2145.9	0.0011	0.998	0.049
	5%	4.3026	18.221	-15.94	2996.3	-0.163	24.37	679.9	0.0004	0.998	0.019
	10%	3.7360	10.956	4.11	1462.0	-0.160	17.57	341.6	0.0003	0.998	0.012
4	1%	5.0052	37.501	-49.78	10659.6	-0.001	32.20	2400.4	0.0010	0.998	0.042
	5%	3.9917	17.658	35.61	3312.6	-0.088	25.69	744.7	0.0004	0.998	0.017
	10%	3.5052	10.932	32.72	1695.6	-0.095	18.64	379.2	0.0003	0.998	0.011
5	1%	4.6578	39.464	-145.06	14334.9	0.056	17.84	2973.2	0.0009	0.999	0.034
	5%	3.7704	18.852	8.38	4670.2	-0.034	18.36	1027.3	0.0004	0.999	0.014
	10%	3.3408	11.843	19.01	2484.1	-0.048	14.30	550.0	0.0003	0.999	0.009
6	1%	4.4025	39.981	-225.90	18881.6	-0.016	39.12	2602.0	0.0009	0.999	0.032
	5%	3.6071	19.078	5.16	6137.1	-0.046	27.02	899.1	0.0004	0.999	0.013
	10%	3.2189	12.170	19.32	3340.5	-0.048	19.25	482.8	0.0002	0.999	0.008
7	1%	4.1964	45.367	-608.38	28419.2	-0.045	34.42	3135.7	0.0010	0.999	0.033
	5%	3.4764	21.309	-120.67	9593.9	-0.055	26.25	1096.2	0.0004	0.999	0.013
	10%	3.1222	13.450	-40.61	5195.8	-0.049	19.23	592.9	0.0003	0.999	0.008
8	1%	4.0382	44.180	-549.34	31594.4	-0.020	40.53	3012.3	0.0009	0.999	0.028
	5%	3.3727	21.403	-107.32	11133.2	-0.036	28.68	1071.3	0.0004	0.999	0.011
	10%	3.0445	13.787	-39.31	6245.6	-0.035	20.37	601.5	0.0002	0.999	0.008
9	1%	3.9048	44.723	-568.27	35781.7	0.018	38.66	2789.0	0.0009	0.999	0.024
	5%	3.2853	22.324	-133.27	13251.1	-0.011	26.93	1005.5	0.0004	0.999	0.010
	10%	2.9792	14.447	-50.31	7473.5	-0.018	19.83	536.9	0.0002	0.999	0.007
10	1%	3.7986	42.582	-375.69	36128.4	0.035	33.27	3466.9	0.0008	0.999	0.020
	5%	3.2145	22.051	-78.77	14078.1	-0.002	24.30	1348.6	0.0003	0.999	0.009
	10%	2.9257	14.493	-23.03	8135.8	-0.009	17.91	778.8	0.0002	0.999	0.006

Note: The RS regression model is equation (D.2). The dependent variable is the simulated α -quantile of the test statistic. Separate regressions are run for each number k of individually $I(0)$ or $I(1)$ variables \mathbf{x}_t in equation (6). $SE(\theta_{0,0})$ denotes the heteroskedasticity-robust standard error of the intercept, \bar{R}^2 the adjusted coefficient of determination, and RMSE the root mean square error.

Table 12: Response surface estimates, F -statistic, case (iv)

k	α	$\theta_{0,0}$	$\theta_{1,0}$	$\theta_{2,0}$	$\theta_{3,0}$	$\theta_{1,1}$	$\theta_{2,1}$	$\theta_{3,1}$	SE($\theta_{0,0}$)	\bar{R}^2	RMSE
I(0)											
0	1%	8.2726	45.413	154.08	1124.1	-2.656	4.29	2511.0	0.0017	0.991	0.136
	5%	6.2605	21.046	65.64	69.9	-2.254	31.51	597.1	0.0008	0.985	0.070
	10%	5.3366	13.098	36.97	-82.9	-1.847	25.94	258.5	0.0006	0.975	0.052
1	1%	6.0697	40.449	-180.48	8097.0	-1.812	42.40	1486.2	0.0013	0.995	0.070
	5%	4.6674	18.888	-72.19	3074.4	-1.501	34.71	312.5	0.0006	0.994	0.035
	10%	4.0162	11.854	-52.40	1862.5	-1.296	25.80	108.9	0.0004	0.990	0.026
2	1%	4.9692	31.697	51.30	6576.4	-1.337	20.93	1756.0	0.0010	0.996	0.056
	5%	3.8699	14.547	82.80	1696.1	-1.169	19.76	493.0	0.0005	0.995	0.026
	10%	3.3556	8.860	71.23	669.4	-1.053	15.83	212.0	0.0004	0.994	0.019
3	1%	4.2898	32.093	-71.62	9641.5	-1.183	2.06	2119.0	0.0008	0.998	0.036
	5%	3.3804	14.797	44.85	2841.1	-1.059	10.87	654.3	0.0003	0.998	0.016
	10%	2.9508	9.222	47.72	1375.5	-0.963	9.88	314.9	0.0002	0.997	0.012
4	1%	3.8394	28.233	48.59	9788.0	-1.121	18.58	1581.3	0.0007	0.998	0.028
	5%	3.0509	13.805	80.96	3052.3	-0.967	13.02	567.5	0.0003	0.999	0.013
	10%	2.6780	8.785	71.64	1461.7	-0.878	9.55	306.0	0.0002	0.998	0.009
5	1%	3.5043	31.201	-197.13	15048.0	-1.052	-1.08	2486.7	0.0007	0.999	0.025
	5%	2.8099	15.343	-16.90	5094.3	-0.902	1.65	1069.2	0.0003	0.999	0.010
	10%	2.4793	9.939	8.12	2741.5	-0.825	1.94	647.6	0.0002	0.999	0.007
6	1%	3.2440	34.974	-518.47	22224.5	-0.956	-13.29	3169.2	0.0008	0.999	0.025
	5%	2.6257	16.655	-119.17	7578.0	-0.839	-3.69	1412.4	0.0003	0.999	0.010
	10%	2.3278	10.885	-54.94	4230.4	-0.773	-2.16	918.3	0.0002	0.999	0.007
7	1%	3.0511	33.952	-524.65	24792.2	-0.936	-16.70	3659.6	0.0006	0.999	0.020
	5%	2.4844	16.891	-154.27	9160.4	-0.818	-6.23	1746.1	0.0003	0.999	0.009
	10%	2.2108	11.225	-82.09	5273.1	-0.751	-4.14	1172.9	0.0002	0.999	0.006
8	1%	2.8925	33.156	-544.74	27841.4	-0.895	-13.89	3907.1	0.0006	0.999	0.018
	5%	2.3687	17.001	-178.26	10697.4	-0.781	-6.90	1987.5	0.0003	0.999	0.008
	10%	2.1153	11.515	-107.24	6391.1	-0.719	-5.24	1384.4	0.0002	0.999	0.006
9	1%	2.7640	31.108	-432.76	28269.9	-0.868	-10.38	4082.1	0.0006	0.999	0.016
	5%	2.2744	16.469	-154.84	11461.3	-0.756	-6.82	2314.2	0.0003	0.999	0.008
	10%	2.0373	11.309	-101.55	7072.7	-0.696	-6.06	1736.0	0.0002	0.999	0.006
10	1%	2.6539	30.619	-409.19	30074.1	-0.825	-18.54	4772.3	0.0006	0.999	0.014
	5%	2.1939	16.723	-171.91	12823.1	-0.728	-10.95	2717.5	0.0003	0.999	0.007
	10%	1.9706	11.816	-135.38	8400.4	-0.672	-9.32	2041.0	0.0002	0.999	0.006
I(1)											
0	1%	8.2726	45.413	154.08	1124.1	-2.656	4.29	2511.0	0.0017	0.991	0.136
	5%	6.2605	21.046	65.64	69.9	-2.254	31.51	597.1	0.0008	0.985	0.070
	10%	5.3366	13.098	36.97	-82.9	-1.847	25.94	258.5	0.0006	0.975	0.052
1	1%	6.6057	49.213	-286.87	9705.8	-0.717	36.21	2279.4	0.0014	0.997	0.072
	5%	5.1415	23.731	-115.45	3583.0	-0.609	29.92	690.4	0.0006	0.996	0.035
	10%	4.4554	15.397	-84.94	2174.8	-0.506	20.79	378.8	0.0004	0.994	0.026
2	1%	5.7472	41.494	13.91	7775.4	-0.121	23.60	2880.7	0.0012	0.997	0.064
	5%	4.5616	20.327	57.22	2120.1	-0.185	20.73	1028.2	0.0005	0.997	0.027
	10%	3.9993	13.130	46.79	903.7	-0.173	14.56	599.0	0.0004	0.997	0.018
3	1%	5.2013	42.313	-108.26	12225.4	-0.141	35.73	2780.6	0.0009	0.998	0.047
	5%	4.1945	20.458	37.75	3664.3	-0.167	30.29	900.6	0.0004	0.998	0.021
	10%	3.7116	13.335	35.17	1883.5	-0.153	22.65	478.8	0.0003	0.998	0.014
4	1%	4.8255	39.532	7.21	13681.4	-0.109	60.74	1974.7	0.0008	0.999	0.036
	5%	3.9383	20.035	73.80	4469.8	-0.115	39.43	596.0	0.0004	0.999	0.016
	10%	3.5111	13.189	59.54	2384.0	-0.106	28.44	282.2	0.0003	0.999	0.011
5	1%	4.5396	42.973	-233.57	20394.7	-0.079	46.84	2749.5	0.0009	0.999	0.035
	5%	3.7464	21.604	-2.07	6891.7	-0.079	32.41	955.1	0.0004	0.999	0.015
	10%	3.3616	14.197	22.15	3730.1	-0.074	24.21	497.3	0.0003	0.999	0.010
6	1%	4.3068	49.285	-669.32	30910.4	0.007	32.96	3384.3	0.0011	0.999	0.035
	5%	3.5944	23.756	-123.77	10370.3	-0.033	27.73	1188.4	0.0004	0.999	0.014
	10%	3.2438	15.485	-38.89	5652.1	-0.042	22.04	618.0	0.0003	0.999	0.009
7	1%	4.1366	49.091	-683.47	35272.2	0.009	27.49	3816.6	0.0009	0.999	0.029
	5%	3.4769	24.468	-154.95	12634.5	-0.028	24.97	1401.5	0.0004	0.999	0.012
	10%	3.1520	16.038	-53.45	6967.6	-0.034	19.97	758.1	0.0002	0.999	0.008
8	1%	3.9994	46.572	-572.10	38234.9	-0.029	47.98	3430.1	0.0009	0.999	0.027
	5%	3.3825	23.668	-104.39	13951.7	-0.041	34.42	1246.2	0.0004	0.999	0.011
	10%	3.0778	15.633	-18.33	7709.4	-0.039	25.81	678.8	0.0002	0.999	0.008
9	1%	3.8817	44.795	-359.86	37839.1	0.003	51.35	2725.5	0.0008	0.999	0.022
	5%	3.3007	23.597	-41.90	14429.5	-0.018	34.84	969.8	0.0004	0.999	0.009
	10%	3.0132	16.091	-6.92	8541.1	-0.020	25.40	526.2	0.0002	0.999	0.007
10	1%	3.7827	43.554	-234.69	39555.6	0.022	43.32	3603.2	0.0008	0.999	0.019
	5%	3.2321	23.516	2.38	15498.7	-0.004	30.06	1443.9	0.0004	0.999	0.009
	10%	2.9597	16.097	26.02	9120.7	-0.010	22.36	832.8	0.0002	0.999	0.006

Note: The RS regression model is equation (D.2). The dependent variable is the simulated α -quantile of the test statistic. Separate regressions are run for each number k of individually $I(0)$ or $I(1)$ variables \mathbf{x}_t in equation (6). SE($\theta_{0,0}$) denotes the heteroskedasticity-robust standard error of the intercept, \bar{R}^2 the adjusted coefficient of determination, and RMSE the root mean square error.

Table 13: Response surface estimates, F -statistic, case (v)

k	α	$\theta_{0,0}$	$\theta_{1,0}$	$\theta_{2,0}$	$\theta_{3,0}$	$\theta_{1,1}$	$\theta_{2,1}$	$\theta_{3,1}$	SE($\theta_{0,0}$)	\bar{R}^2	RMSE
I(0)											
0	1%	15.6672	74.372	185.49	1121.9	-8.703	87.05	3197.9	0.0041	0.976	0.327
	5%	11.6378	31.535	31.60	229.5	-7.003	98.25	346.7	0.0021	0.942	0.174
	10%	9.7837	17.880	-1.41	123.6	-5.744	68.01	-4.3	0.0015	0.894	0.125
1	1%	8.6578	53.977	-386.80	11734.0	-3.184	72.38	1673.8	0.0021	0.992	0.115
	5%	6.5535	23.744	-188.44	4752.8	-2.584	51.94	271.9	0.0010	0.985	0.061
	10%	5.5742	14.234	-145.42	3046.8	-2.227	35.98	73.9	0.0007	0.972	0.046
2	1%	6.3327	35.190	126.81	6172.7	-1.852	30.34	1984.7	0.0015	0.994	0.077
	5%	4.8627	14.956	118.92	1272.0	-1.599	25.70	524.5	0.0007	0.991	0.039
	10%	4.1747	8.269	97.39	290.0	-1.435	20.04	208.0	0.0005	0.986	0.028
3	1%	5.1477	33.218	-7.67	9634.1	-1.585	25.23	1833.6	0.0010	0.997	0.048
	5%	4.0046	14.191	82.85	2586.3	-1.356	24.63	412.6	0.0004	0.996	0.023
	10%	3.4649	8.117	75.53	1137.6	-1.215	20.17	111.4	0.0003	0.994	0.017
4	1%	4.4331	28.756	100.06	9858.5	-1.374	38.23	1147.5	0.0008	0.998	0.034
	5%	3.4833	13.035	118.45	2787.8	-1.151	24.95	241.6	0.0004	0.998	0.016
	10%	3.0333	7.755	98.15	1222.3	-1.029	18.30	40.4	0.0003	0.997	0.011
5	1%	3.9439	31.261	-148.35	15056.8	-1.226	14.56	2086.4	0.0008	0.999	0.029
	5%	3.1298	14.459	22.61	4762.9	-1.022	11.20	751.8	0.0003	0.999	0.012
	10%	2.7420	8.810	40.12	2417.6	-0.922	9.44	379.2	0.0002	0.999	0.009
6	1%	3.5836	34.980	-478.71	22258.5	-1.072	-1.42	2768.6	0.0009	0.999	0.027
	5%	2.8731	15.844	-86.21	7309.6	-0.921	4.56	1078.1	0.0003	0.999	0.011
	10%	2.5309	9.862	-25.60	3913.4	-0.836	4.13	638.2	0.0002	0.999	0.007
7	1%	3.3228	34.048	-499.60	24964.6	-1.016	-8.14	3293.5	0.0007	0.999	0.022
	5%	2.6822	16.125	-121.73	8835.6	-0.868	-0.70	1450.6	0.0003	0.999	0.009
	10%	2.3730	10.338	-57.36	4993.3	-0.790	0.58	903.9	0.0002	0.999	0.006
8	1%	3.1167	32.784	-499.57	27638.0	-0.968	-2.71	3415.5	0.0007	0.999	0.019
	5%	2.5318	16.062	-140.03	10303.2	-0.823	0.13	1630.4	0.0003	0.999	0.008
	10%	2.2488	10.504	-74.45	5983.0	-0.749	0.19	1082.3	0.0002	0.999	0.006
9	1%	2.9520	30.647	-381.70	27841.5	-0.918	-1.10	3521.8	0.0006	0.999	0.016
	5%	2.4111	15.672	-118.12	10965.9	-0.785	-0.25	1845.2	0.0003	0.999	0.007
	10%	2.1491	10.421	-67.38	6526.9	-0.716	-0.88	1330.0	0.0002	0.999	0.005
10	1%	2.8148	29.849	-334.35	29037.9	-0.867	-9.39	4222.0	0.0006	0.999	0.014
	5%	2.3108	15.822	-126.64	12126.2	-0.749	-4.91	2283.5	0.0003	0.999	0.007
	10%	2.0663	10.804	-93.09	7703.5	-0.686	-3.89	1639.1	0.0002	0.999	0.005
I(1)											
0	1%	15.6672	74.372	185.49	1121.9	-8.703	87.05	3197.9	0.0041	0.976	0.327
	5%	11.6378	31.535	31.60	229.5	-7.003	98.25	346.7	0.0021	0.942	0.174
	10%	9.7837	17.880	-1.41	123.6	-5.744	68.01	-4.3	0.0015	0.894	0.125
1	1%	9.4757	66.489	-508.98	13578.6	-1.643	54.31	2765.4	0.0021	0.995	0.116
	5%	7.2736	31.008	-258.62	5500.5	-1.410	38.54	804.6	0.0010	0.991	0.062
	10%	6.2405	19.323	-195.54	3484.2	-1.229	22.90	471.9	0.0007	0.982	0.049
2	1%	7.3794	49.423	14.60	8348.6	-0.346	20.79	3530.4	0.0016	0.997	0.080
	5%	5.7921	23.183	50.86	2084.2	-0.427	16.13	1328.6	0.0007	0.996	0.036
	10%	5.0387	14.418	30.44	884.8	-0.417	8.50	828.5	0.0005	0.994	0.026
3	1%	6.2934	47.128	-98.49	12772.6	-0.322	43.34	3073.3	0.0011	0.998	0.055
	5%	5.0280	22.005	30.71	3780.0	-0.343	33.73	991.0	0.0005	0.998	0.024
	10%	4.4209	13.666	28.22	1876.3	-0.319	23.69	545.6	0.0004	0.997	0.017
4	1%	5.6231	43.203	-2.49	14546.8	-0.220	67.63	2136.5	0.0010	0.998	0.041
	5%	4.5535	21.223	53.59	4805.8	-0.222	41.27	675.8	0.0004	0.999	0.018
	10%	4.0376	13.548	34.80	2644.2	-0.210	28.11	360.6	0.0003	0.998	0.012
5	1%	5.1587	45.567	-235.69	21158.0	-0.168	54.40	2814.2	0.0010	0.999	0.039
	5%	4.2277	22.329	-19.03	7209.9	-0.156	34.58	994.8	0.0004	0.999	0.016
	10%	3.7757	14.209	3.89	3939.3	-0.149	24.74	534.9	0.0003	0.999	0.011
6	1%	4.8057	51.615	-691.45	31924.2	-0.058	39.12	3419.8	0.0011	0.999	0.038
	5%	3.9860	24.135	-135.46	10634.7	-0.091	30.11	1191.8	0.0004	0.999	0.015
	10%	3.5826	15.234	-50.44	5789.7	-0.100	22.94	629.4	0.0003	0.999	0.010
7	1%	4.5507	51.076	-714.13	36444.1	-0.034	30.83	3916.8	0.0010	0.999	0.031
	5%	3.8043	24.729	-167.90	12906.1	-0.068	25.95	1427.3	0.0004	0.999	0.012
	10%	3.4363	15.807	-70.96	7194.4	-0.074	19.74	792.0	0.0003	0.999	0.008
8	1%	4.3518	48.107	-597.71	39355.8	-0.058	49.73	3573.6	0.0009	0.999	0.028
	5%	3.6625	23.853	-122.07	14288.0	-0.071	34.41	1301.0	0.0004	0.999	0.012
	10%	3.3218	15.342	-35.71	7944.8	-0.070	25.04	722.4	0.0003	0.999	0.008
9	1%	4.1866	46.029	-378.60	38771.0	-0.021	53.53	2803.7	0.0009	0.999	0.023
	5%	3.5444	23.693	-62.05	14837.0	-0.043	34.68	1031.3	0.0004	0.999	0.010
	10%	3.2264	15.744	-27.02	8844.7	-0.045	24.54	586.4	0.0003	0.999	0.007
10	1%	4.0495	45.024	-287.62	41126.0	0.005	43.67	3743.9	0.0008	0.999	0.020
	5%	3.4469	23.628	-27.22	16086.0	-0.025	29.75	1506.3	0.0004	0.999	0.009
	10%	3.1484	15.668	8.66	9388.1	-0.031	21.73	873.8	0.0003	0.999	0.006

Note: The RS regression model is equation (D.2). The dependent variable is the simulated α -quantile of the test statistic. Separate regressions are run for each number k of individually $I(0)$ or $I(1)$ variables \mathbf{x}_k in equation (6). SE($\theta_{0,0}$) denotes the heteroskedasticity-robust standard error of the intercept, \bar{R}^2 the adjusted coefficient of determination, and RMSE the root mean square error.

Table 14: Response surface estimates, t -statistic, case (i)

k	α	$\theta_{0,0}$	$\theta_{1,0}$	$\theta_{2,0}$	$\theta_{3,0}$	$\theta_{1,1}$	$\theta_{2,1}$	$\theta_{3,1}$	SE($\theta_{0,0}$)	\bar{R}^2	RMSE
I(0)											
0	1%	-2.5570	-4.228	17.84	-177.5	0.536	-12.54	1.3	0.0003	0.977	0.015
	5%	-1.9356	-1.794	27.57	-257.0	0.477	-11.72	63.6	0.0002	0.919	0.008
	10%	-1.6133	-0.889	28.87	-263.2	0.427	-10.13	63.4	0.0001	0.812	0.006
1	1%	-2.5601	-3.931	13.74	-347.2	0.267	-14.04	12.1	0.0003	0.983	0.014
	5%	-1.9372	-1.621	25.65	-335.6	0.243	-9.16	52.5	0.0001	0.947	0.007
	10%	-1.6145	-0.751	29.32	-332.2	0.237	-6.68	47.1	0.0001	0.758	0.006
2	1%	-2.5597	-4.083	26.05	-691.1	0.162	-12.93	-11.4	0.0003	0.984	0.014
	5%	-1.9373	-1.682	32.07	-480.2	0.163	-7.05	28.6	0.0002	0.948	0.007
	10%	-1.6145	-0.773	34.74	-425.3	0.170	-4.09	17.9	0.0001	0.773	0.006
3	1%	-2.5618	-3.574	2.53	-629.1	0.136	-14.66	35.5	0.0003	0.981	0.013
	5%	-1.9386	-1.385	19.98	-401.7	0.137	-6.99	40.5	0.0002	0.939	0.007
	10%	-1.6157	-0.498	26.13	-351.6	0.150	-3.60	20.5	0.0001	0.781	0.005
4	1%	-2.5614	-3.616	0.82	-770.8	0.092	-11.73	-39.5	0.0003	0.981	0.013
	5%	-1.9387	-1.409	22.33	-486.7	0.115	-5.63	15.0	0.0002	0.938	0.007
	10%	-1.6160	-0.499	32.12	-464.8	0.139	-2.94	10.3	0.0001	0.809	0.005
5	1%	-2.5637	-3.065	-31.75	-536.9	0.094	-13.35	8.0	0.0004	0.979	0.012
	5%	-1.9393	-1.264	14.86	-443.1	0.106	-5.17	18.0	0.0002	0.927	0.007
	10%	-1.6162	-0.472	34.07	-507.5	0.128	-1.80	-3.5	0.0002	0.797	0.005
6	1%	-2.5630	-3.175	-30.45	-776.8	0.072	-11.72	-30.6	0.0004	0.980	0.012
	5%	-1.9396	-1.196	12.88	-486.8	0.097	-4.65	14.3	0.0002	0.933	0.007
	10%	-1.6168	-0.354	33.32	-533.1	0.126	-1.74	5.1	0.0002	0.828	0.005
7	1%	-2.5635	-3.220	-27.84	-1090.6	0.074	-12.67	-12.1	0.0004	0.982	0.012
	5%	-1.9402	-1.073	11.31	-571.5	0.098	-5.69	50.1	0.0002	0.938	0.007
	10%	-1.6173	-0.202	31.79	-557.9	0.124	-2.02	25.2	0.0002	0.832	0.005
8	1%	-2.5631	-3.528	-7.83	-1696.0	0.067	-11.43	-68.4	0.0004	0.984	0.013
	5%	-1.9402	-1.173	20.72	-803.3	0.091	-4.42	15.7	0.0002	0.946	0.007
	10%	-1.6173	-0.196	37.97	-689.4	0.118	-0.96	3.5	0.0002	0.845	0.005
9	1%	-2.5637	-3.277	-22.08	-1847.4	0.064	-12.25	-50.4	0.0004	0.981	0.012
	5%	-1.9407	-1.082	19.38	-892.4	0.091	-4.62	26.0	0.0002	0.935	0.007
	10%	-1.6177	-0.179	45.41	-855.5	0.120	-0.99	3.4	0.0002	0.854	0.005
10	1%	-2.5639	-3.316	-18.55	-2274.8	0.062	-12.64	7.4	0.0005	0.977	0.012
	5%	-1.9410	-0.999	18.75	-1000.6	0.090	-5.32	81.1	0.0002	0.918	0.007
	10%	-1.6179	-0.151	52.67	-1031.8	0.120	-1.32	38.4	0.0002	0.838	0.005
I(1)											
0	1%	-2.5570	-4.228	17.84	-177.5	0.536	-12.54	1.3	0.0003	0.977	0.015
	5%	-1.9356	-1.794	27.57	-257.0	0.477	-11.72	63.6	0.0002	0.919	0.008
	10%	-1.6133	-0.889	28.87	-263.2	0.427	-10.13	63.4	0.0001	0.812	0.006
1	1%	-3.2084	-6.088	13.10	-388.0	0.341	-13.90	-53.9	0.0003	0.989	0.016
	5%	-2.5919	-2.736	22.11	-338.2	0.336	-9.88	44.5	0.0002	0.971	0.009
	10%	-2.2631	-1.555	23.72	-314.9	0.333	-7.78	61.0	0.0001	0.905	0.008
2	1%	-3.6158	-8.125	50.03	-994.4	0.263	-6.47	-294.8	0.0003	0.992	0.016
	5%	-3.0024	-3.498	39.12	-541.5	0.317	-2.75	-127.3	0.0002	0.974	0.010
	10%	-2.6728	-1.854	35.56	-401.7	0.352	-0.78	-92.8	0.0002	0.924	0.010
3	1%	-3.9436	-7.563	-15.28	-449.8	0.361	-11.84	-294.7	0.0003	0.993	0.015
	5%	-3.3268	-2.848	6.46	-130.7	0.416	-3.31	-206.9	0.0002	0.975	0.010
	10%	-2.9950	-1.143	16.79	-73.3	0.461	0.17	-202.6	0.0002	0.946	0.010
4	1%	-4.2179	-8.454	14.11	-1172.9	0.455	-14.42	-383.7	0.0004	0.994	0.015
	5%	-3.6006	-2.813	28.79	-528.0	0.550	-7.95	-209.3	0.0002	0.976	0.011
	10%	-3.2672	-0.772	39.91	-411.0	0.613	-5.05	-183.6	0.0002	0.956	0.012
5	1%	-4.4577	-9.614	60.84	-2104.3	0.507	-10.49	-613.0	0.0004	0.992	0.016
	5%	-3.8367	-3.803	109.24	-1648.5	0.599	-1.01	-499.2	0.0003	0.962	0.013
	10%	-3.5015	-1.583	130.40	-1590.4	0.665	2.85	-498.5	0.0004	0.947	0.014
6	1%	-4.6757	-10.385	116.71	-3354.8	0.587	-10.28	-829.7	0.0005	0.992	0.017
	5%	-4.0551	-3.525	150.65	-2381.0	0.721	-4.64	-600.1	0.0004	0.965	0.014
	10%	-3.7195	-0.854	167.29	-2184.4	0.806	-2.55	-554.3	0.0004	0.959	0.015
7	1%	-4.8779	-10.502	156.73	-4534.7	0.679	-13.75	-980.6	0.0005	0.992	0.017
	5%	-4.2556	-2.905	183.40	-3046.5	0.815	-7.00	-733.8	0.0004	0.962	0.016
	10%	-3.9191	0.112	198.56	-2717.8	0.904	-4.72	-686.2	0.0004	0.963	0.016
8	1%	-5.0635	-11.403	249.18	-6523.7	0.731	-7.50	-1453.6	0.0006	0.992	0.019
	5%	-4.4408	-2.487	231.22	-3900.0	0.875	-1.77	-1128.6	0.0005	0.960	0.017
	10%	-4.1036	0.922	240.35	-3336.5	0.972	0.05	-1065.4	0.0005	0.965	0.019
9	1%	-5.2413	-11.169	303.31	-8191.6	0.831	-12.18	-1646.7	0.0006	0.990	0.020
	5%	-4.6169	-1.830	292.46	-5127.8	0.992	-7.77	-1198.8	0.0006	0.961	0.019
	10%	-4.2789	1.662	315.56	-4623.3	1.098	-6.40	-1112.7	0.0007	0.968	0.021
10	1%	-5.4088	-10.274	331.14	-9654.3	0.883	-7.08	-2554.5	0.0007	0.986	0.020
	5%	-4.7837	-0.179	321.40	-6140.7	1.058	-4.90	-2101.8	0.0007	0.962	0.020
	10%	-4.4453	3.781	337.64	-5412.7	1.173	-5.17	-2005.1	0.0008	0.971	0.021

Note: The RS regression model is equation (D.2). The dependent variable is the simulated α -quantile of the test statistic. Separate regressions are run for each number k of individually $I(0)$ or $I(1)$ variables \mathbf{x}_t in equation (6). SE($\theta_{0,0}$) denotes the heteroskedasticity-robust standard error of the intercept, \bar{R}^2 the adjusted coefficient of determination, and RMSE the root mean square error.

Table 15: Response surface estimates, t -statistic, case (iii)

k	α	$\theta_{0,0}$	$\theta_{1,0}$	$\theta_{2,0}$	$\theta_{3,0}$	$\theta_{1,1}$	$\theta_{2,1}$	$\theta_{3,1}$	SE($\theta_{0,0}$)	\bar{R}^2	RMSE
I(0)											
0	1%	-3.4298	-6.418	-32.65	332.8	0.683	-3.74	-270.9	0.0003	0.982	0.023
	5%	-2.8619	-2.902	-10.94	158.3	0.671	-6.05	-77.9	0.0002	0.948	0.015
	10%	-2.5672	-1.666	-3.56	77.2	0.625	-5.09	-31.3	0.0001	0.900	0.012
1	1%	-3.4290	-6.987	24.00	-853.0	0.539	-17.17	-52.3	0.0003	0.989	0.018
	5%	-2.8609	-3.076	24.74	-531.0	0.540	-10.99	36.4	0.0002	0.965	0.012
	10%	-2.5663	-1.667	25.81	-441.9	0.544	-7.87	45.4	0.0002	0.932	0.010
2	1%	-3.4266	-6.721	1.31	-559.3	0.343	-2.29	-388.9	0.0004	0.989	0.017
	5%	-2.8595	-2.620	3.01	-158.8	0.411	0.27	-191.6	0.0002	0.972	0.010
	10%	-2.5653	-1.147	8.09	-104.4	0.454	1.56	-138.8	0.0002	0.960	0.008
3	1%	-3.4302	-5.918	-8.97	-921.4	0.410	-13.32	-146.2	0.0003	0.991	0.014
	5%	-2.8623	-1.795	-9.54	-204.4	0.461	-6.22	-42.5	0.0002	0.979	0.008
	10%	-2.5679	-0.265	-5.45	-53.7	0.502	-3.39	-21.3	0.0001	0.976	0.007
4	1%	-3.4308	-5.528	-11.93	-1222.7	0.402	-13.59	-152.3	0.0003	0.990	0.014
	5%	-2.8630	-1.488	-0.16	-475.9	0.472	-7.04	-20.5	0.0002	0.982	0.008
	10%	-2.5687	0.087	5.42	-277.9	0.521	-4.10	1.0	0.0002	0.985	0.006
5	1%	-3.4302	-5.498	-2.50	-1689.0	0.388	-11.04	-195.9	0.0004	0.992	0.012
	5%	-2.8625	-1.280	8.71	-711.6	0.463	-5.29	-34.5	0.0002	0.984	0.007
	10%	-2.5683	0.442	12.29	-427.6	0.514	-2.84	2.6	0.0002	0.986	0.006
6	1%	-3.4304	-5.296	1.52	-2132.2	0.387	-10.69	-190.7	0.0004	0.992	0.012
	5%	-2.8626	-1.066	17.75	-948.2	0.461	-3.33	-66.5	0.0002	0.987	0.007
	10%	-2.5684	0.673	24.53	-619.3	0.514	-0.28	-44.6	0.0002	0.989	0.006
7	1%	-3.4299	-5.292	27.71	-3022.0	0.387	-10.41	-239.7	0.0004	0.993	0.012
	5%	-2.8626	-0.622	20.17	-1159.7	0.457	-2.94	-74.4	0.0002	0.986	0.007
	10%	-2.5685	1.295	21.42	-665.1	0.511	-0.26	-34.8	0.0002	0.989	0.005
8	1%	-3.4301	-5.097	37.53	-3660.4	0.382	-8.30	-293.3	0.0004	0.991	0.012
	5%	-2.8627	-0.419	32.93	-1475.8	0.456	0.07	-168.2	0.0002	0.986	0.007
	10%	-2.5686	1.533	36.16	-905.1	0.510	3.29	-140.4	0.0002	0.990	0.006
9	1%	-3.4300	-5.032	53.67	-4395.3	0.381	-6.21	-369.7	0.0004	0.990	0.012
	5%	-2.8633	-0.064	42.72	-1811.6	0.463	1.77	-284.3	0.0002	0.986	0.007
	10%	-2.5694	1.994	43.55	-1081.3	0.520	4.52	-230.9	0.0002	0.990	0.006
10	1%	-3.4314	-4.262	22.34	-4335.8	0.388	-7.73	-299.3	0.0005	0.987	0.012
	5%	-2.8644	0.640	28.19	-1730.2	0.475	-1.75	-81.1	0.0003	0.984	0.007
	10%	-2.5701	2.596	41.47	-1115.0	0.533	1.45	-55.3	0.0002	0.990	0.006
I(1)											
0	1%	-3.4298	-6.418	-32.65	332.8	0.683	-3.74	-270.9	0.0003	0.982	0.023
	5%	-2.8619	-2.902	-10.94	158.3	0.671	-6.05	-77.9	0.0002	0.948	0.015
	10%	-2.5672	-1.666	-3.56	77.2	0.625	-5.09	-31.3	0.0001	0.900	0.012
1	1%	-3.7946	-8.954	39.61	-1093.4	0.527	-19.61	-62.8	0.0004	0.990	0.020
	5%	-3.2140	-4.244	36.61	-684.5	0.517	-12.19	36.5	0.0002	0.968	0.014
	10%	-2.9080	-2.556	38.55	-594.4	0.523	-8.70	47.6	0.0002	0.915	0.013
2	1%	-4.0902	-10.288	25.15	-835.1	0.336	0.21	-588.7	0.0004	0.993	0.019
	5%	-3.5031	-4.818	30.89	-401.7	0.420	3.00	-332.0	0.0002	0.976	0.014
	10%	-3.1906	-2.804	36.87	-327.2	0.474	4.65	-266.7	0.0002	0.936	0.013
3	1%	-4.3540	-10.586	19.00	-1163.3	0.471	-9.57	-522.6	0.0004	0.995	0.017
	5%	-3.7596	-4.441	19.67	-283.1	0.541	-1.04	-357.6	0.0003	0.979	0.013
	10%	-3.4423	-2.066	23.42	-65.9	0.598	2.57	-333.3	0.0003	0.950	0.013
4	1%	-4.5868	-11.348	56.77	-2131.2	0.565	-13.23	-622.0	0.0004	0.995	0.016
	5%	-3.9877	-4.855	77.50	-1176.9	0.674	-5.09	-421.7	0.0003	0.979	0.013
	10%	-3.6664	-2.453	97.35	-1068.5	0.748	-1.39	-397.2	0.0003	0.961	0.014
5	1%	-4.7974	-12.275	112.58	-3378.9	0.633	-13.40	-803.0	0.0004	0.995	0.017
	5%	-4.1950	-4.786	125.13	-2009.3	0.766	-8.44	-493.3	0.0004	0.974	0.014
	10%	-3.8713	-1.912	140.27	-1752.3	0.856	-6.58	-420.6	0.0004	0.954	0.015
6	1%	-4.9901	-13.760	212.04	-5299.6	0.681	-5.99	-1243.5	0.0005	0.994	0.018
	5%	-4.3843	-5.767	230.97	-3542.9	0.816	2.52	-1013.7	0.0005	0.970	0.017
	10%	-4.0581	-2.706	253.14	-3243.9	0.909	5.70	-978.1	0.0005	0.960	0.018
7	1%	-5.1719	-14.050	276.98	-7038.5	0.747	-6.57	-1545.9	0.0005	0.994	0.018
	5%	-4.5637	-4.831	256.85	-4194.2	0.880	1.42	-1232.6	0.0005	0.963	0.017
	10%	-4.2357	-1.279	267.61	-3585.7	0.973	4.34	-1176.7	0.0005	0.957	0.018
8	1%	-5.3435	-14.516	356.92	-9008.6	0.798	2.77	-2236.6	0.0006	0.991	0.020
	5%	-4.7320	-5.272	374.13	-6160.3	0.944	13.00	-2025.8	0.0006	0.963	0.019
	10%	-4.4025	-1.489	389.40	-5457.4	1.046	15.96	-1990.3	0.0006	0.965	0.021
9	1%	-5.5100	-13.404	385.52	-10621.9	0.889	3.11	-3208.2	0.0007	0.990	0.020
	5%	-4.8970	-3.385	399.12	-7200.6	1.054	12.52	-3104.8	0.0007	0.969	0.019
	10%	-4.5665	0.748	417.04	-6421.4	1.166	15.13	-3144.6	0.0007	0.973	0.021
10	1%	-5.6635	-14.155	537.48	-14372.1	0.970	-2.52	-3136.1	0.0009	0.985	0.022
	5%	-5.0495	-3.069	521.85	-9850.3	1.154	1.38	-2695.3	0.0008	0.962	0.022
	10%	-4.7175	1.126	560.01	-9287.0	1.275	1.94	-2595.8	0.0009	0.970	0.023

Note: The RS regression model is equation (D.2). The dependent variable is the simulated α -quantile of the test statistic. Separate regressions are run for each number k of individually $I(0)$ or $I(1)$ variables \mathbf{x}_k in equation (6). SE($\theta_{0,0}$) denotes the heteroskedasticity-robust standard error of the intercept, \bar{R}^2 the adjusted coefficient of determination, and RMSE the root mean square error.

Table 16: Response surface estimates, t -statistic, case (v)

k	α	$\theta_{0,0}$	$\theta_{1,0}$	$\theta_{2,0}$	$\theta_{3,0}$	$\theta_{1,1}$	$\theta_{2,1}$	$\theta_{3,1}$	$SE(\theta_{0,0})$	\bar{R}^2	RMSE
$I(0)$											
0	1%	-3.9594	-9.262	-16.02	-72.9	1.187	-20.52	-220.4	0.0005	0.978	0.036
	5%	-3.4117	-4.616	-2.02	-19.4	1.050	-16.42	-22.2	0.0003	0.944	0.024
	10%	-3.1280	-2.862	0.71	-3.5	0.929	-11.23	1.2	0.0002	0.898	0.019
1	1%	-3.9525	-10.789	91.02	-1992.6	0.763	-19.87	-192.9	0.0004	0.987	0.025
	5%	-3.4064	-5.322	66.45	-1137.3	0.743	-11.24	-60.1	0.0003	0.965	0.018
	10%	-3.1237	-3.233	57.56	-856.0	0.733	-6.57	-44.6	0.0002	0.935	0.015
2	1%	-3.9567	-8.224	-33.20	-563.7	0.588	-9.72	-426.3	0.0004	0.990	0.021
	5%	-3.4091	-3.358	-17.09	-37.8	0.635	-2.92	-227.2	0.0002	0.974	0.014
	10%	-3.1259	-1.510	-7.47	62.7	0.668	0.63	-185.5	0.0002	0.963	0.012
3	1%	-3.9600	-7.446	-30.31	-1196.7	0.659	-20.37	-201.9	0.0004	0.992	0.017
	5%	-3.4121	-2.419	-18.03	-307.4	0.706	-11.26	-43.7	0.0002	0.982	0.011
	10%	-3.1288	-0.458	-9.78	-96.7	0.743	-7.16	-13.2	0.0002	0.977	0.009
4	1%	-3.9589	-7.344	-9.52	-1960.8	0.636	-20.39	-109.1	0.0004	0.993	0.015
	5%	-3.4112	-2.317	9.20	-834.6	0.699	-9.10	-38.7	0.0002	0.989	0.009
	10%	-3.1281	-0.368	23.37	-607.6	0.747	-4.28	-40.6	0.0002	0.990	0.008
5	1%	-3.9589	-7.155	11.82	-2778.2	0.638	-18.94	-156.5	0.0004	0.994	0.014
	5%	-3.4119	-1.604	8.85	-1031.5	0.704	-9.23	-8.6	0.0002	0.990	0.008
	10%	-3.1287	0.492	19.96	-661.3	0.754	-4.65	3.8	0.0002	0.991	0.007
6	1%	-3.9584	-7.034	38.21	-3730.7	0.630	-17.16	-186.0	0.0004	0.995	0.013
	5%	-3.4121	-1.180	24.21	-1435.6	0.711	-8.13	-18.0	0.0002	0.993	0.007
	10%	-3.1295	1.118	29.16	-861.8	0.769	-4.06	6.1	0.0002	0.993	0.006
7	1%	-3.9575	-6.939	63.56	-4671.5	0.607	-11.15	-349.3	0.0004	0.994	0.013
	5%	-3.4113	-0.744	35.01	-1754.3	0.691	-3.64	-109.3	0.0002	0.991	0.007
	10%	-3.1285	1.662	39.30	-1050.0	0.747	0.37	-76.8	0.0002	0.993	0.006
8	1%	-3.9581	-6.365	69.77	-5435.3	0.608	-9.92	-416.4	0.0004	0.993	0.012
	5%	-3.4114	-0.427	60.01	-2328.1	0.692	0.17	-246.5	0.0002	0.992	0.007
	10%	-3.1286	2.044	64.10	-1472.5	0.750	4.62	-223.6	0.0002	0.994	0.006
9	1%	-3.9587	-5.852	73.44	-6090.6	0.613	-9.47	-403.1	0.0005	0.992	0.012
	5%	-3.4122	0.126	75.17	-2792.7	0.703	1.26	-328.0	0.0003	0.992	0.007
	10%	-3.1295	2.642	83.23	-1877.5	0.764	6.09	-348.5	0.0002	0.994	0.007
10	1%	-3.9602	-4.887	48.23	-6285.7	0.623	-11.39	-298.2	0.0005	0.989	0.012
	5%	-3.4131	0.980	70.39	-2937.1	0.718	-1.93	-134.1	0.0003	0.991	0.007
	10%	-3.1305	3.549	82.88	-1981.9	0.783	2.01	-106.7	0.0003	0.994	0.007
$I(1)$											
0	1%	-3.9594	-9.262	-16.02	-72.9	1.187	-20.52	-220.4	0.0005	0.978	0.036
	5%	-3.4117	-4.616	-2.02	-19.4	1.050	-16.42	-22.2	0.0003	0.944	0.024
	10%	-3.1280	-2.862	0.71	-3.5	0.929	-11.23	1.2	0.0002	0.898	0.019
1	1%	-4.2423	-12.982	107.28	-2216.0	0.673	-17.64	-275.4	0.0004	0.989	0.027
	5%	-3.6829	-6.773	80.14	-1266.2	0.654	-8.56	-123.6	0.0003	0.970	0.019
	10%	-3.3893	-4.445	73.88	-999.1	0.655	-3.71	-107.6	0.0002	0.932	0.017
2	1%	-4.4946	-12.270	4.22	-983.3	0.496	-2.92	-719.4	0.0004	0.993	0.022
	5%	-3.9239	-6.180	23.99	-360.5	0.559	5.01	-486.2	0.0003	0.979	0.016
	10%	-3.6222	-3.862	37.59	-269.6	0.611	8.67	-435.5	0.0002	0.951	0.016
3	1%	-4.7214	-12.803	22.32	-1718.9	0.623	-12.39	-699.5	0.0004	0.995	0.019
	5%	-4.1427	-6.012	46.11	-792.1	0.702	-3.23	-464.3	0.0003	0.983	0.014
	10%	-3.8352	-3.368	59.42	-570.4	0.764	1.06	-421.5	0.0003	0.958	0.014
4	1%	-4.9232	-14.522	117.59	-3548.8	0.663	-9.17	-957.2	0.0004	0.995	0.017
	5%	-4.3380	-7.476	163.07	-2451.3	0.767	3.86	-847.6	0.0003	0.982	0.015
	10%	-4.0260	-4.703	188.60	-2259.9	0.845	9.50	-861.9	0.0004	0.969	0.015
5	1%	-5.1123	-15.288	181.92	-5110.2	0.735	-9.81	-1176.4	0.0005	0.995	0.018
	5%	-4.5230	-7.181	207.53	-3285.3	0.861	-0.09	-919.4	0.0004	0.979	0.016
	10%	-4.2074	-3.989	229.31	-2918.5	0.948	4.18	-886.2	0.0004	0.960	0.017
6	1%	-5.2898	-16.099	275.37	-7277.1	0.817	-10.54	-1474.8	0.0005	0.996	0.018
	5%	-4.6979	-6.718	262.66	-4341.6	0.969	-2.54	-1149.5	0.0005	0.979	0.017
	10%	-4.3801	-3.056	275.64	-3679.5	1.071	0.78	-1096.3	0.0005	0.967	0.018
7	1%	-5.4562	-16.433	347.30	-9217.5	0.862	-7.31	-1871.5	0.0006	0.994	0.019
	5%	-4.8601	-6.246	326.82	-5622.3	1.020	-1.69	-1432.3	0.0005	0.968	0.018
	10%	-4.5397	-2.227	339.39	-4803.6	1.125	0.88	-1354.9	0.0006	0.961	0.019
8	1%	-5.6130	-17.509	469.67	-11915.9	0.893	6.31	-2687.0	0.0007	0.990	0.021
	5%	-5.0123	-7.505	495.74	-8385.3	1.048	17.95	-2426.2	0.0007	0.964	0.022
	10%	-4.6894	-3.462	524.69	-7615.2	1.157	21.92	-2387.6	0.0007	0.965	0.023
9	1%	-5.7677	-16.761	551.99	-14837.3	0.981	7.39	-3806.7	0.0008	0.989	0.021
	5%	-5.1644	-6.340	607.68	-11312.1	1.152	20.63	-3819.9	0.0008	0.971	0.021
	10%	-4.8399	-2.040	648.28	-10633.6	1.269	24.58	-3881.2	0.0008	0.973	0.023
10	1%	-5.9121	-17.343	717.56	-19266.1	1.059	2.13	-3774.9	0.0010	0.984	0.023
	5%	-5.3060	-6.425	791.19	-15498.8	1.250	9.72	-3433.1	0.0010	0.964	0.024
	10%	-4.9800	-1.800	834.06	-14708.0	1.378	10.12	-3284.1	0.0011	0.969	0.025

Note: The RS regression model is equation (D.2). The dependent variable is the simulated α -quantile of the test statistic. Separate regressions are run for each number k of individually $I(0)$ or $I(1)$ variables \mathbf{x}_k in equation (6). $SE(\theta_{0,0})$ denotes the heteroskedasticity-robust standard error of the intercept, \bar{R}^2 the adjusted coefficient of determination, and RMSE the root mean square error.