Response surface regressions for critical value bounds and approximate p-values in equilibrium correction models*

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Abstract

Single-equation conditional equilibrium correction models can be used to test for the existence of a level relationship among the variables of interest. The distributions of the respective test statistics are nonstandard under the null hypothesis of no such relationship and critical values need to be obtained with stochastic simulations. We run response surface regressions based on more than 95 billion F-statistics and 57 billion t-statistics to obtain precise finite-sample critical values and approximate p-values for the Pesaran, Shin, and Smith (2001, *Journal of Applied Econometrics* 16: 289–326) bounds test. Our estimates allow to compute critical value bounds and approximate p-values for any sample size, number of variables, and lag order. Response surfaces for the augmented Dickey-Fuller unit-root test statistics result as special cases.

Keywords: Equilibrium correction model; Unit roots; Cointegration; Bounds test; Level relationship; Response surface regression; Critical values, Approximate p-values

JEL Classification: C12; C15; C32; C46; C63

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1 Introduction

The empirical analysis of time series data is often confronted with test statistics that have nonstandard distributions in the presence of a unit root. While the asymptotic distributions can be characterized as functions of stochastic processes such as Brownian motions, the corresponding quantiles that are needed to compute critical values for hypothesis testing are usually obtained with stochastic simulations. As an additional complication, the distributions of the test statistics generally depend on the specific assumptions about the data-generating process and the specification of the estimated model, in particular whether an intercept or time trend are allowed. In a multivariate environment, the dimension of the variable space and the cointegration rank matter. All of these remarks apply to the Pesaran et al. (2001) bounds test for the existence of a level relationship in an unrestricted conditional equilibrium correction model that is the focus of our paper.

In finite samples, the distributions of the test statistics may depend on further characteristics of the estimation. While appending the regression model with additional stationary variables does not affect the asymptotic distributions of unit-root and cointegration tests, their influence on the finite-sample distributions might be nonnegligible. Applied researchers are confronted with a large number of empirically relevant scenarios that give rise to possibly different distributions. The tabulation of critical values quickly approaches space limits and is usually done only for a selected number of practically relevant situations. This leaves blank areas in the possibility space that can be interpolated only to a limited extent. In this paper, we provide response surface estimates to systematically fill these blank spots for the Pesaran et al. (2001) critical value bounds. Our results are applicable for any sample size and lag augmentation, several cases regarding the deterministic model components, and in particular without a limit on the number of variables in the level relationship.

The response surface technique has been introduced into the field of unit-root testing and cointegration analysis by MacKinnon (1991) for a range of Dickey and Fuller (1979) and Engle and Granger (1987) tests. Ericsson and MacKinnon (2002) provide response surface estimates for the cointegration t-statistic in single-equation conditional error correction models that comprise the Dickey-Fuller statistic as a special case. Both asymptotic and finite-sample critical values can be easily obtained from these estimates.[1] However, they do not take the influence of the lag order on the finite-sample critical values into account. In contrast, for the augmented Dickey-Fuller unit-root test, the response surface regressions of Cheung and Lai (1995a) allow to compute appropriate finite-sample critical values for any lag order.[2] Cook (2001) compares the response surfaces from Cheung and Lai (1995a) with those from MacKinnon (1991) and concludes that adjusting the critical values for the lag order leads to a gain in power.

As a complement to the generalized Dickey-Fuller t-statistic, Pesaran et al. (2001) propose a related F-statistic to test for the existence of a level relationship in a conditional equilibrium correction model.[3] They derive the asymptotic distributions of both test statistics under the null hypothesis of no level relationship. The distributions not only depend on the deterministic model components and the number of variables but also on the unknown cointegration rank. Pesaran et al. (2001) recommend a bounds testing procedure that yields conclusive results if the observed value of the test statistic falls outside of the critical-value bounds established for the situations where all long-run forcing variables are purely integrated of either order zero, $I(0)$, or order one, $I(1)$. Because the bounds procedure does not require that all variables are individually $I(1)$, the considered concept of a level relationship is broader than that of cointegration.[4]

Pesaran et al. (2001) use stochastic simulations to compute near-asymptotic critical values for situations with up to 10 long-run forcing variables and for five different cases regarding the restrictions on the deterministic model components. However, the asymptotic distribution might be a poor approximation of the finite-sample distributions for empirically relevant small sample sizes. Finite-sample critical values are tabulated by Mills and Pentecost (2001), Narayan and Smyth (2004), Kanioura and Turner (2005), and Narayan (2005), but they cover only a limited area in the possibility space. In addition, the pre-

[1]Previously tabulated critical values for a small set of sample sizes can be found in Fuller (1976) and Dickey (1976) for the univariate and Banerjee et al. (1998) for the multivariate setting.

[2]Response surface estimates for finite-sample critical values of other unit-root tests are provided by Cheung and Lai (1995b), Harvey and van Dijk (2006), Otero and Baum (2017), and Otero and Smith (2012, 2017). All of them take the lag order into account. Further related applications of the response surface methodology include Sephton (1995, 2008, 2017), Carrion-i-Silvestre et al. (1999), and Presno and López (2003).

[3]In the univariate case, this statistic reduces to the Dickey and Fuller (1981) unit-root F-statistic.

[4]McNown et al. (2018) propose a bootstrap procedure for the Pesaran et al. (2001) test that allows for conclusive inference when the test statistic falls within the two bounds.

cision of these critical values suffers from a relatively small number of replications in the respective simulations. In the case of Narayan (2005), this becomes apparent because the tabulated values do not comply with the monotonic decline of the actual response surface toward the asymptotic critical value. Turner (2006) applies the response surface methodology of MacKinnon (1991) for the F-statistic but again only for a narrow subset of the empirically relevant situations.

In this paper, we provide response surface estimates for the Pesaran et al. (2001) bounds test. We compute more than 95 billion F-statistics and 57 billion t-statistics in our stochastic simulations for all five cases regarding the deterministic model components, different variable counts, various lag orders, and a wide range of sample sizes. While previously reported critical values cannot easily be extrapolated beyond the largest number of variables considered in the respective simulations, our response surface estimates extend for any number of variables in the level relationship. Not surprisingly, our simulations and estimates predict a diminishing influence on the critical values of adding another variable to the model. The accuracy of our predictions confirms that it is not necessary to run separate response surface regressions for each variable count.

Last but not least, MacKinnon (1994, 1996) extends the response surface methodology to numerically approximate p-values and distribution functions.[5] We adopt his approach and find that it works very well for the test statistics considered in this paper. The predicted critical values from our response surface regressions and the procedure to obtain approximate p-values are implemented in the *Stata* program provided by Kripfganz and Schneider (2018).[6]

2 Bounds testing for the existence of a level relationship

This section provides a compact summary of the model and assumptions used by Pesaran et al. (2001) to derive the asymptotic distributions of their bounds testing procedure for the existence of a level relationship.

[5] MacKinnon et al. (1999) proceed along the same lines for cointegration tests in a vector error correction model.

[6] The program can be installed from `http://www.kripfganz.de/stata/`.

2.1 Equilibrium correction model

Let $\mathbf{z}_t$ be a column vector of $k+1$ random variables, generated by a vector-autoregressive (VAR) model of order q:

$$\mathbf{\Phi}(L)(\mathbf{z}_t - \mathbf{b}_0 - \mathbf{b}_1 t) = \boldsymbol{\epsilon}_t, \quad t = q+1, q+2, \ldots, T, \tag{1}$$

where $\mathbf{\Phi}(L) = \mathbf{I}_{k+1} - \sum_{i=1}^{q} \mathbf{\Phi}_i L^i$ is a q-th order polynomial in the lag operator L with unknown $(k+1) \times (k+1)$ coefficient matrices $\mathbf{\Phi}_i$, and $\mathbf{b}_0$ and $\mathbf{b}_1$ are $(k+1)$-dimensional vectors of unknown intercept and trend parameters. The initial observations $\mathbf{z}_1, \mathbf{z}_2, \ldots, \mathbf{z}_q$ are assumed to be observed. By defining the long-run multiplier matrix $\mathbf{\Pi} = \sum_{i=1}^{q} \mathbf{\Phi}_i - \mathbf{I}_{k+1}$ and the short-run coefficient matrices $\mathbf{\Gamma}_i = -\sum_{j=i+1}^{q} \mathbf{\Phi}_j$, $i = 1, 2, \ldots, q-1$, we can rewrite the above VAR(q) model in vector equilibrium correction (VEC) form:

$$\Delta \mathbf{z}_t = \mathbf{a}_0 + \mathbf{a}_1 t + \mathbf{\Pi} \mathbf{z}_{t-1} + \sum_{i=1}^{q-1} \mathbf{\Gamma}_i \Delta \mathbf{z}_{t-i} + \boldsymbol{\epsilon}_t, \tag{2}$$

where $\Delta = (1 - L)$ is the first-difference operator, $\mathbf{a}_0 = -\mathbf{\Pi}\mathbf{b}_0 + (\mathbf{\Pi} + \mathbf{\Gamma})\mathbf{b}_1$, $\mathbf{a}_1 = -\mathbf{\Pi}\mathbf{b}_1$, and $\mathbf{\Gamma} = \mathbf{I}_{k+1} - \sum_{i-1}^{q-1} \mathbf{\Gamma}_i$. Let us partition $\mathbf{z}_t = (y_t, \mathbf{x}_t')'$ and the long-run multiplier matrix conformably as

$$\mathbf{\Pi} = \begin{pmatrix} \pi_{yy} & \boldsymbol{\pi}_{yx}' \\ \boldsymbol{\pi}_{xy} & \mathbf{\Pi}_{xx} \end{pmatrix}.$$

Furthermore, partition $\mathbf{\Gamma}_i = (\boldsymbol{\gamma}_{yi}, \mathbf{\Gamma}_{xi}')'$ and $\mathbf{\Gamma} = (\boldsymbol{\gamma}_y, \mathbf{\Gamma}_x')'$.

In analogy to Pesaran et al. (2001), we make the following assumptions:

Assumption 1: The roots of $|\mathbf{I}_{K+1} - \sum_{i=1}^{q} \mathbf{\Phi}_i z^i| = 0$ satisfy $-1 < 1/z \leq 1$. The data-generating process of $\mathbf{z}_t$ is integrated at most of order unity.[7]

Assumption 2: The vector of errors $\boldsymbol{\epsilon}_t$ is independent multivariate normally distributed, $\boldsymbol{\epsilon}_t \sim \mathcal{N}(\mathbf{0}, \mathbf{\Omega})$, with mean vector zero and positive-definite variance matrix $\mathbf{\Omega}$.

Assumption 3: The data-generating process of $\mathbf{x}_t$ is long-run forcing for the process of y_t, that is $\boldsymbol{\pi}_{xy} = \mathbf{0}$.

Assumption 4: The matrix $\mathbf{\Pi}_{xx}$ has rank r with $0 \leq r \leq k$.

[7] See Pesaran et al. (2001) for a more formal statement of the last part of this assumption.

Assumption 1 allows the individual elements of the vector $\mathbf{z}_t$ to be $I(0)$ or $I(1)$, or to be cointegrated. The cointegration order for the data-generating process of $\mathbf{x}_t$ is defined by Assumption 4. Consequently, the rank of the long-run multiplier matrix $\mathbf{\Pi}$ is either r or $r+1$. Assumption 3 implies that $\mathbf{\Pi}$ being of rank r corresponds to the parameter restriction $\pi_{yy} = 0$, while the rank $r+1$ necessitates $\pi_{yy} \neq 0$. Under Assumptions 3 and 4, we can express the long-run multiplier matrix as $\mathbf{\Pi} = \boldsymbol{\alpha}_y \boldsymbol{\beta}_y' + \mathbf{A}\mathbf{B}'$, where $\boldsymbol{\alpha}_y = (\alpha_{yy}, \mathbf{0}')'$ and $\boldsymbol{\beta}_y = (\beta_{yy}, \boldsymbol{\beta}_{yx}')'$ are $(k+1)$-dimensional vectors, and $\mathbf{A} = (\boldsymbol{\alpha}_{yx}, \mathbf{A}_{xx}')'$ and $\mathbf{B} = (\mathbf{0}, \mathbf{B}_{xx}')'$ are $(k+1) \times r$ matrices of full column rank, respectively.[8] With the normalization $\beta_{yy} = 1$, it follows $\pi_{yy} = \alpha_{yy}$. Clearly, $\mathbf{A}\mathbf{B}' = \mathbf{0}$ if $r = 0$.

Under Assumptions 2 and 3, we can now obtain the following equilibrium correction (EC) model for y_t conditional on $\mathbf{x}_t$ and their past values $\mathbf{z}_1, \mathbf{z}_2, \ldots, \mathbf{z}_{t-1}$:

$$\Delta y_t = c_0 + c_1 t + \boldsymbol{\pi}' \mathbf{z}_{t-1} + \sum_{i=1}^{q-1} \boldsymbol{\psi}_i' \Delta \mathbf{z}_{t-i} + \boldsymbol{\omega}' \Delta \mathbf{x}_t + u_t, \tag{3}$$

with intercept $c_0 = -\boldsymbol{\pi}'\mathbf{b}_0 + [(\boldsymbol{\gamma}_y - \mathbf{\Gamma}_x'\boldsymbol{\omega})' + \boldsymbol{\pi}']\mathbf{b}_1$ and trend coefficient $c_1 = -\boldsymbol{\pi}'\mathbf{b}_1$, and where $\boldsymbol{\pi} = (\pi_{yy}, \boldsymbol{\varphi}')'$, with $\boldsymbol{\varphi} = \boldsymbol{\pi}_{yx} - \mathbf{\Pi}_{xx}'\boldsymbol{\omega}$. Furthermore, $\boldsymbol{\psi}_i = \boldsymbol{\gamma}_{yi} - \mathbf{\Gamma}_{xi}'\boldsymbol{\omega}$ for all i. With the partition of the error term $\boldsymbol{\epsilon}_t = (\epsilon_{yt}, \boldsymbol{\epsilon}_{xt}')'$ and the conformably partitioned variance matrix

$$\mathbf{\Omega} = \begin{pmatrix} \omega_{yy} & \boldsymbol{\omega}_{xy}' \\ \boldsymbol{\omega}_{xy} & \mathbf{\Omega}_{xx} \end{pmatrix},$$

$\boldsymbol{\omega} = \mathbf{\Omega}_{xx}^{-1}\boldsymbol{\omega}_{xy}$ is obtained as the coefficient vector in the linear projection of ϵ_{yt} on $\boldsymbol{\epsilon}_{xt}$. The corresponding projection error u_t is independent multivariate normally distributed under Assumption 2, $u_t \sim \mathcal{N}(\mathbf{0}, \omega_{yy} - \boldsymbol{\omega}_{xy}'\mathbf{\Omega}_{xx}^{-1}\boldsymbol{\omega}_{xy})$.

A conditional level relationship between y_t and $\mathbf{x}_t$ exists if both $\pi_{yy} \neq 0$ and $\boldsymbol{\varphi} \neq \mathbf{0}$, irrespective of whether y_t is $I(0)$ or $I(1)$. In the second case, the data-generating processes of y_t and $\mathbf{x}_t$ are cointegrated. In the opposite situation, $\boldsymbol{\pi} = \mathbf{0}$, the conditional EC model (3) only contains first-differenced terms such that no level relationship between y_t and $\mathbf{x}_t$ can exist and y_t must be $I(1)$. There are two degenerate cases. If just $\pi_{yy} = 0$, y_t is still $I(1)$ and there exists only a level relationship among the elements of $\mathbf{x}_t$ not involving y_t. If

[8] This expression of the long-run multiplier matrix is useful for the derivation of the asymptotic distribution of the t-statistic used by Banerjee et al. (1998) to test whether $\pi_{yy} = 0$. See Pesaran et al. (2001) for details.

π_{yy} is the only nonzero element of $\boldsymbol{\pi}$, y_t is generated by a trend-stationary or $I(0)$ process not involving the levels of $\mathbf{x}_t$.

2.2 Bounds test

In the light of the two degenerate situations, the following successive testing procedure can be applied:

(1) Test the joint null hypothesis $H_0^{\boldsymbol{\pi}} : \boldsymbol{\pi} = \mathbf{0}$ versus the alternative hypothesis $H_1^{\boldsymbol{\pi}} : \boldsymbol{\pi} \neq \mathbf{0}$.

(2) If $H_0^{\boldsymbol{\pi}}$ is rejected, test the single hypothesis $H_0^{\pi_{yy}} : \pi_{yy} = 0$ versus $H_1^{\pi_{yy}} : \pi_{yy} < 0$, under the additional assumption that either $r = 0$ or $\boldsymbol{\alpha}_{yx} - \mathbf{A}_{xx}'\boldsymbol{\omega} = \mathbf{0}$ if $0 < r \leq k$.

(3) If $H_0^{\pi_{yy}}$ is rejected, test the joint hypothesis $H_0^{\boldsymbol{\theta}} : \boldsymbol{\theta} = \mathbf{0}$ versus $H_1^{\boldsymbol{\theta}} : \boldsymbol{\theta} \neq \mathbf{0}$, where $\boldsymbol{\theta} = -\boldsymbol{\varphi}/\pi_{yy}$ are the long-run multipliers in the conditional level relationship between y_t and $\mathbf{x}_t$.

The reason for proceeding with steps (2) and (3) is that the alternative hypothesis $H_1^{\boldsymbol{\pi}}$ in step (1) does not rule out any of the two degenerate cases mentioned above. The latter are the subject of the hypothesis tests in steps (2) and (3). Only if all three null hypotheses are rejected, we can conclude that there is statistical evidence for the existence of a nondegenerate level relationship between y_t and $\mathbf{x}_t$.

As demonstrated by Pesaran et al. (2001), y_t is $I(1)$ under the null hypothesis in steps (1) and (2) and the respective test statistics have nonstandard asymptotic distributions. The additional assumption required for step (2) implies $\boldsymbol{\varphi} = \pi_{yy}\boldsymbol{\beta}_{yx}$. Consequently, under $H_0^{\pi_{yy}}$ we have again $\boldsymbol{\pi} = \mathbf{0}$ as in step (1), but $H_1^{\pi_{yy}}$ is more informative at the cost of imposing additional structure on the data-generating process. Without this assumption, the asymptotic distribution of the t-statistic would depend on nuisance parameters and tabulations of critical values for general purposes would become practically infeasible.[9]

For the long-run multipliers $\boldsymbol{\theta}$ that are the subject of step (3), Pesaran and Shin (1998) and Hassler and Wolters (2006) show that the ordinary least squares (OLS) estimator is super-consistent if $\mathbf{x}_t$ contains $I(1)$ regressors, and it is asymptotically normally distributed

[9]See Pesaran et al. (2001) for a discussion. Banerjee et al. (1998) assume $r = 0$ and briefly argue that the critical values obtained under this assumption will lead to a conservative test if it is violated.

irrespective of the order of integration. The remainder of this text is therefore primarily concerned with the test statistics in steps (1) and (2).[10]

As we have seen above, the restricted VAR formulation (1) imposes constraints on the coefficients c_0 and c_1 in the conditional EC model (3) that ensure that the cointegration rank r does not affect the deterministic trending behavior.[11] Pesaran et al. (2001) distinguish five cases, depending on which deterministic components are included in the model specification and whether we disregard the implied restrictions on their coefficients or not:

(i) No intercept and no trend are included, $c_0 = c_1 = 0$,

(ii) A restricted intercept is included but no trend, $c_0 = -\boldsymbol{\pi}'\mathbf{b}_0$ and $c_1 = 0$,

(iii) An unrestricted intercept is included but no trend, $c_0 \neq 0$ and $c_1 = 0$,

(iv) An unrestricted intercept and a restricted trend are included, $c_0 \neq 0$ and $c_1 = -\boldsymbol{\pi}'\mathbf{b}_1$,

(v) An unrestricted intercept and an unrestricted trend are included, $c_0 \neq 0$ and $c_1 \neq 0$.

As emphasized by Pesaran et al. (2001), the data-generating processes under case (ii) and (iii) are identical, and similarly for cases (iv) and (v), but the Wald test statistics in step (1) and their asymptotic distributions differ under the null hypothesis $H_0^{\boldsymbol{\pi}}$. For the single hypothesis test in step (2), the restrictions can be ignored.

Pesaran et al. (2001) argue that the critical values for the two polar cases of $\mathbf{x}_t$ being purely $I(0)$ or purely $I(1)$ provide lower and upper bounds, respectively, when the orders of integration and the cointegration rank r are unknown. They derive the asymptotic distributions of the Wald test statistic in step (1) and the t-statistic in step (2), respectively. Both statistics are functions of standard Brownian motions, possibly de-meaned and de-trended, and depend on the cointegration rank r.[12]

Remark 1: Hassler (2000) discusses the consequences of $\mathbf{b}_1 = (0, \mathbf{b}_{1x}')'$ with $\mathbf{b}_{1x} \neq \mathbf{0}$ in the data-generating process (1) for the t-test in step (2) given a cointegration rank $r = 0$. If the data is not de-trended, that is case (iii), the tabulated critical values are not directly applicable because the linear time trend dominates one of the stochastic trends. Instead,

[10] McNown et al. (2018) propose a bootstrap procedure for the inference on the coefficients $\boldsymbol{\varphi}$ of the level regressors.

[11] See Pesaran et al. (2000) for details.

[12] See Theorems 3.1 and 3.2 in Pesaran et al. (2001).

the correct critical values would be those from case (v) given a cointegration rank $r = 1$. While these critical values are not explicitly tabulated, Hassler (2000) demonstrates that this corresponds to a situation with $k-1$ long-run forcing regressors and cointegration rank $r = 0$. This argumentation does not directly extend to step (1) because the dimension of the coefficient vector $\boldsymbol{\pi}$ is still $1 + k$. We conjecture that the critical values for the F-test under $r = 1$ cannot simply be taken from the tabulations for $k - 1$ regressors under $r = 0$. It is thus advisable to de-trend the variables, that is to estimate the model directly under case (iv) or (v), when a linear time trend is suspected in the data-generating process.

3 Critical values and approximate p-values

Pesaran et al. (2001) use stochastic simulations to obtain near-asymptotic critical value bounds based on a sample size of 1000 time periods for the F-statistic under $H_0^{\boldsymbol{\pi}}$ in step (1) and the t-statistic under $H_0^{\pi_{yy}}$ in step (2).[13] They tabulate the critical values for the range of $k \in [0, 10]$ long-run forcing variables. Several other authors provide finite-sample critical values for a subset of the relevant situations. We summarize the existing literature in Table 1.[14] A number of authors tabulated critical values for selected sample sizes that require interpolations between the reported sample sizes. Accordingly, they are unanimously superseded by the estimates from response surface regressions, whenever the latter are available and sufficiently precise.

Although unit-root tests are not the primary focus of our work, the Dickey-Fuller test statistics result as a special case in the univariate setting, $k = 0$. When there is no need for a lag augmentation, the response surface estimates of MacKinnon (1996, 2010) and Ericsson and MacKinnon (2002) are the primary source for accurate finite-sample critical values, as far as the t-statistic is concerned. In many situations, however, serial correlation in the error term threatens to undermine the validity of the test results. A remedy is the augmented Dickey-Fuller test based on a higher-order autoregressive model. The test statistic remains the same, and Said and Dickey (1984) prove that its asymptotic distribution is unaffected as well. However, the degrees-of-freedom reduction

[13]The F-statistic is obtained by dividing the Wald statistic by $k + 1$ in cases (i), (iii), and (v), and by $k + 2$ in cases (ii) and (iv).

[14]The distributions of the cointegration test statistics resulting from the Engle and Granger (1987) two-stage procedure differ from those considered in the Pesaran et al. (2001) framework. Corresponding response surface estimates can be found in MacKinnon (1991, 1996, 2010).

Table 1: Critical value tabulations in the previous literature

	$T-q$	q	k	$I(d)$	F cases	t cases^{+}
Fuller (1976)	$25, 50, 100, 250, 500, \infty$	1	0	–	–	(i), (iii), (v)
Dickey (1976)	$25, 50, 100, 250, 500, 750, \infty$	1	0	–	–	(i), (iii), (v)
Dickey and Fuller (1981)	$25, 50, 100, 250, 500, \infty$	1	0	–	(ii), (iv)	–
MacKinnon (1991, 2010)	response surface	1	0	–	–	(i), (iii), (v)
Cheung and Lai (1995a)	response surface	≥ 1	0	–	–	(i), (iii), (v)
MacKinnon (1996)*	response surface	1	0	–	–	(i), (iii), (v)
Banerjee et al. (1998)	$25, 50, 100, 500, \infty$	1	$[1, 5]$	1	–	(iii), (v)
Pesaran et al. (2001)	1000	0	$[0, 10]$	$0, 1$	(i)–(v)	(i), (iii), (v)
Mills and Pentecost (2001)	$22, 26$	1	3	$0, 1$	(i)–(v)	(i), (iii), (v)
Ericsson and MacKinnon (2002)*	response surface	1	$[0, 11]$	1	–	(i), (iii), (v)
Narayan and Smyth (2004)	$22, 25, 30, 37$	0	2	$0, 1$	(ii)	–
Kanioura and Turner (2005)**	$50, 100, 200, 500$	0/1	$[1, 3]$	1	(iii)	(i)
Narayan (2005)	30–80 in steps of 5	0	$[0, 7]$	$0, 1$	(ii)–(v)	–
Turner (2006)	response surface	1	$[1, 3]$	$0, 1$	(iii), (v)	–

Note: The regression model used by these authors to compute the F-statistics and t-statistics can be written as in equation (6) with q lags and k long-run forcing variables that are integrated of order d. For the unit-root tests, i.e. $k = 0$, the specifications are equivalent for $q = 0$ and $q = 1$.
*MacKinnon (1996) and Ericsson and MacKinnon (2002) provide computer programs that compute the critical values and approximate p-values.
**Kanioura and Turner (2005) compute their test statistics from different regression specifications. Their F-statistic is based on $q = 1$ and their t-statistic on $q = 0$. The latter is only tabulated for $k = 1$.
$^{+}$MacKinnon (1996, 2010) and Ericsson and MacKinnon (2002) furthermore consider the t-statistic in the presence of a quadratic trend.

affects the finite-sample distributions. The response surfaces from Cheung and Lai (1995a) provide a more accurate prediction of the critical values in that situation. An even better fit is obtained with our estimates in Section 3.2 due to the substantially larger number of replications. For the unit-root F-statistic, we are the first to provide comprehensive response surface estimates.[15]

In the multivariate setting, the lag order dependence of finite-sample critical values has been neglected completely so far. A stronger emphasis has been put on the number of long-run forcing variables. The response surface estimates from Ericsson and MacKinnon (2002) cover the cointegration t-statistic for up to 11 long-run forcing variables that are purely $I(1)$. For the F-statistic, the coverage is much thinner. To date, only Turner (2006) provides such response surface estimates, but merely for cases (iii) and (v) and a small number of up to 3 long-run forcing variables. We fill the gaps left by the existing literature

[15] Dickey and Fuller (1981) tabulate a few critical values for the restricted intercept or trend cases (ii) and (iv). While the F-statistic in the unrestricted cases (i), (iii), and (v) equals the square of the t-statistic, this is not true for the quantiles of the corresponding distributions. Consequently, separate critical values need to be obtained.

with our response surface regressions in Section 3.3. Based on these new estimates, critical values can be computed for any sample size, lag order, and number of variables, differentiating between all five cases regarding the deterministic model components. Moreover, a more informative statistical inference is possible with the approximate p-values that can be computed based on the methodology proposed by MacKinnon (1994, 1996).

3.1 Monte Carlo simulations

For each replication in our Monte Carlo simulations, we generate the data according to the following processes that satisfy $H_0^{\boldsymbol{\pi}}$ and $H_0^{\pi_{yy}}$:

$$y_t = y_{t-1} + \epsilon_{yt}, \tag{4}$$

$$\mathbf{x}_t = \mathbf{P}\mathbf{x}_{t-1} + \boldsymbol{\epsilon}_{xt}, \tag{5}$$

for $t = 1, 2, \ldots, T + 50$ and with the initializations $y_0 = 0$ and $\mathbf{x}_0 = \mathbf{0}$. The first 50 observations are discarded. The elements of the vector of shocks $\boldsymbol{\epsilon}_t$ are independently drawn from the standard normal distribution. The coefficient matrix $\mathbf{P}$ equals either the zero or the identity matrix, depending on whether $\mathbf{x}_t$ is supposed to be purely $I(0)$ or $I(1)$.[16]

The test statistics are constructed from the unrestricted regression coefficients in a reparameterization of equation (3):

$$\Delta y_t = c_0 + c_1 t + \pi_{yy} y_{t-1} + \boldsymbol{\varphi}' \mathbf{x}_t + \sum_{i=1}^{q-1} \psi_{yi} \Delta y_{t-i} + \sum_{i=0}^{q-1} \boldsymbol{\psi}'_{xi} \Delta \mathbf{x}_{t-i} + u_t, \tag{6}$$

where $(\psi_{yi}, \boldsymbol{\psi}'_{xi})' = \boldsymbol{\psi}_i$ for all $i = 0, 1, \ldots, q-1$. The use of the contemporaneous $\mathbf{x}_t$ instead of the lagged $\mathbf{x}_{t-1}$ is advocated by Pesaran and Shin (1998). It has the advantage that the short-run coefficients $\boldsymbol{\psi}_{xi}$ can be treated as unrestricted for all lag orders q, while in the representation (3) the presence of the term $\boldsymbol{\omega}' \Delta \mathbf{x}_t$ induces an overparameterization when $q = 0$.[17] In cases (i), (iii), and (v), under the null hypothesis $H_0^{\boldsymbol{\pi}}$, the F-statistic is used to test for joint insignificance of the coefficients π_{yy} and $\boldsymbol{\varphi}$ in the regression (6). In

[16] The data-generating process is identical to the one used by Pesaran et al. (2001), besides the discarded observations.

[17] The lag specification $q = 0$ can be obtained from the VAR(1) model in equation (1) by imposing the restriction $\boldsymbol{\omega} = \boldsymbol{\varphi}$.

cases (ii) and (iv), the respective restriction on the intercept c_0 or trend coefficient c_1 is added. Under $H_0^{\pi_{yy}}$, the t-statistic is computed for π_{yy}.

For each of the 2 integration orders and 5 deterministic model component cases, we run separate simulations for all combinations of $k \in [0, 10]$,

$$T \in \{18, 20, 22, 25, 28, 30, 32, 36, 40, 50, 60, 80, 100, 150, 200, 300, 400, 500, 1000\},$$

and $q \in \{0, 1, 2, 3, 4, 6, 8, 12\}$, subject to the restriction that there are at least twice as many observations as coefficients in equation (6) to ensure a sufficient number of degrees of freedom for the tests.[18] The effective sample size is $T - \max(q, 1)$. This yields a total of 9,528 simulation designs.[19] For each design, we run 100,000 replications and we repeat the procedure another 100 times, which we refer to as 'meta replications'. We thus compute a total number of 9.528×10^{10} F-statistics and 5.744×10^{10} t-statistics.[20]

In order to be able to store such a large number of statistics, we first round the statistics to three digits after the decimal point and then apply a suitable transformation that significantly reduces storage requirements.[21] The effect of rounding on the response surface regressions is absolutely negligible.

3.2 Separate response surface regressions for each k

For each meta replication and simulation design, we compute the quantiles of interest from the simulated distributions of both test statistics. In the next step, separate response surfaces are estimated for each quadruplet $\{c, k, d, p\}$, where c is the case regarding the deterministic model components, k is the number of long-run forcing variables with integration order d, and p is the level of the quantile. Given the 100 meta replications, up to 19 choices of the time horizon T, and 8 different lag orders q, we have between 5,900

[18] That is $\max(1, q) + k(q + 1) + \mathcal{I}(c_0 \neq 0) + \mathcal{I}(c_1 \neq 0) \leq (T - \max(q, 1))/2$, where $\mathcal{I}(\cdot)$ is an indicator function that equals unity if the respective deterministic component is included and zero otherwise. In addition, $q = 0$ is not relevant for $k = 0$.

[19] There are 1,960 simulation designs for case (i), 1,910 designs for cases (ii) and (iii) each, and 1,874 designs for cases (iv) and (v), respectively.

[20] There is no longer a computational reason as in MacKinnon (1996) for the use of meta replications instead of a single experiment with 10 million replications. His second argument, that meta replications provide an easy way to evaluate the experimental randomness, survives.

[21] Details on the compression procedure as well as other computational aspects are relegated to Appendix A.

and 12,400 observations per estimation.[22]

We follow the conventional practice of regressing the simulated quantiles on a polynomial in the inverse effective sample size $N(T,q) = T - \max(q,1)$. To account for the influence of the lag order, we add interaction terms between the number of unrestricted short-run coefficients, $H(q,k)$, and the negative powers of the effective sample size. When all variables have the same lag order q without zero restrictions, as in our experiments, then $H(q,k) = \max(q-1,0) + kq$. The response surface model thus becomes

$$Q_k(T,q) = \sum_{j=0}^{m} \sum_{l=0}^{n} \theta_{j,l} [N(T,q)]^{-j} [H(q,k)]^{l} + u, \tag{7}$$

where $Q_k(T,q)$ is the respective quantile from each meta replication for a given k. The presence of stationary first-differenced terms in equation (6) when $q > 0$ does not affect the asymptotic properties of the distribution which implies the restrictions $\theta_{0,l} = 0$ for all $l > 0$. The intercept $\theta_{0,0}$ can then be interpreted as the asymptotic quantile when $T \to \infty$. There is no clear guidance for the choice of the polynomial orders m and n, and the optimal order possibly differs across the many regressions. As emphasized by MacKinnon (1996), it is important to choose the same specification across quantiles in order to avoid discontinuities in the distributions that are inferred from the predicted values. After extensive experimentation, we have chosen $m = 3$ and $n = 1$. The latter provides a better fit than alternatively setting $n = 3$ together with the restrictions $\theta_{j,l} = 0$ whenever $j \neq l$ for $l > 0$, which has been done by Cheung and Lai (1995a). Equation (7) thus reduces to

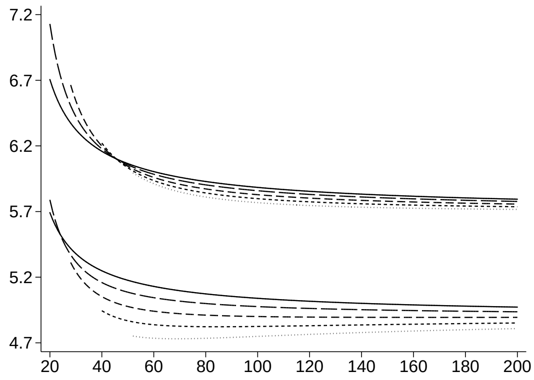
$$Q_k(T,q) = \theta_{0,0} + \sum_{j=1}^{3} \theta_{j,0} \frac{1}{[N(T,q)]^{j}} + \sum_{j=1}^{3} \theta_{j,1} \frac{H(q,k)}{[N(T,q)]^{j}} + u. \tag{8}$$

In Appendix B, we report the ordinary least squares results for the quantiles corresponding to a size of 1%, 5%, and 10%.[23] Tables 2 to 17 also contain the standard error (SE) of the intercept, robust to heteroskedasticity,[24] as a measure of uncertainty about

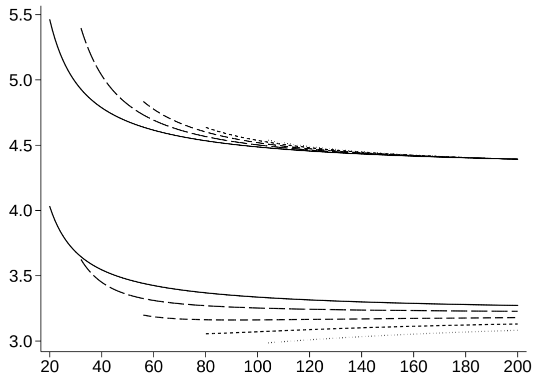
[22] The largest number of observations is available for $k = 1$ in case (i), and the smallest number for $k = 10$ in cases (iv) and (v).

[23] Estimates for other quantiles are available upon request.

[24] The error variance is a decreasing function in the effective sample size which could be taken into account with a generalized least squares procedure as proposed by MacKinnon (1991) or a generalized method of moments estimator as discussed by MacKinnon (1994, 1996). However, the numerical differences in the



(a) $k = 1$



(b) $k = 3$

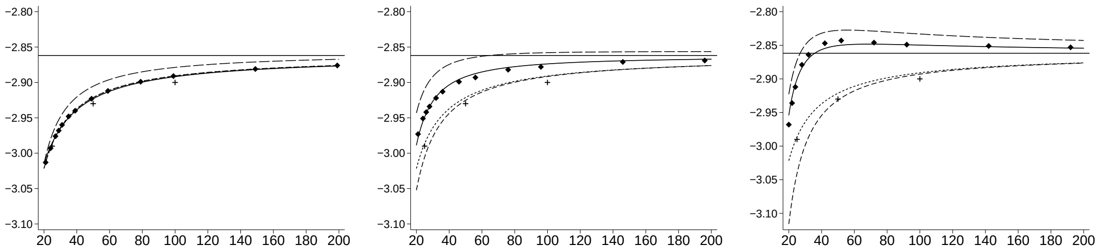
Figure 1: Response surfaces from equation (8) for the F-statistic in case (iii) with $k \in \{1, 3\}$ variables at the 5% significance level for selected lag orders $q \in \{0, 3, 6, 9, 12\}$ over a range of effective sample sizes $T - \max(q, 1)$. The solid curves represent the lower bound (closer to zero) and the upper bound for $q = 0$. With increasing lag order, the curves have shorter dashes.

the asymptotic quantile. It is always smaller than 0.0041 for the F-statistic and below 0.0011 for the t-statistic. In most experimental designs, the standard error remains far below this magnitude. However, the reported standard errors are too small because they are conditional on the correct specification of the response surface model, as emphasized by MacKinnon (1991).

The asymptotic critical values can be read off directly from the response surface intercept $\theta_{0,0}$. Our estimates are close to the corresponding near-asymptotic critical values tabulated by Pesaran et al. (2001). The absolute difference is for the most part below 0.05, both for the F-statistic and the t-statistic. However, these asymptotic critical values are less useful in small samples. For a given number of variables in the level relationship, finite-sample critical values can be computed from the regression coefficients for any combination of the effective sample size and number of short-run coefficients.

Previously reported critical values do not take the lag augmentation in equation (6) into account and might thus be inaccurate in many empirically relevant situations, in particular when the sample size is relatively small. Figure 1 highlights for the F-statistic that the predicted critical values from our response surface regressions not only vary with

predictions are negligible, in particular given the remaining model uncertainty about the correct functional form of the response surface regressions.



(a) $q = 1$ (b) $q = 4$ (c) $q = 8$

Figure 2: Response surfaces from equation (8) for the t-statistic in case (iii) with $k = 0$ variables at the 5% significance level for selected lag orders q over a range of effective sample sizes $T - \max(q, 1)$. The diamonds are the critical values computed from the simulated distribution of the 10^7 t-statistics. The horizontal line represents the respective estimate of $\theta_{0,0}$ in Table 14 and the solid curve the corresponding response surface. The long-dashed curve is the response surface from Cheung and Lai (1995a), the medium-dashed curve from Ericsson and MacKinnon (2002), and the short-dashed curve from MacKinnon (2010). Crosses are tabulated critical values from Dickey (1976).

the sample size but also with the lag order q. This is particularly true for the lower bound critical values that exhibit a slower convergence rate to the respective asymptotic critical value than the upper bounds. Moreover, the convexity of the response surface increases with the lag order. While the slope of the response surface is negative in q for larger sample sizes, it can become positive for relatively small sample sizes, increasingly so the more long-run forcing variables are in the model.

For $k = 0$, there is obviously no distinction possible between $I(0)$ and $I(1)$ variables in the level relationship, and the respective response surfaces coincide. In this situation, the F-statistic in cases (ii) and (iv) is the one analyzed by Dickey and Fuller (1981). In cases (i), (iii), and (v), it equals the square of the t-statistic. The latter corresponds to the familiar augmented Dickey-Fuller unit-root test statistic. The asymptotic critical values obtained from our response surface regressions closely match those reported in the previous literature.

Response surface estimates for the original Dickey and Fuller (1979) test statistic, $q = 1$, have been previously obtained by MacKinnon (1991, 2010) and Ericsson and MacKinnon (2002).[25] Cheung and Lai (1995a) go one step further by estimating a response surface that allows the quantiles of the distribution to vary with the lag order. Figure 2

[25] Dickey (1976) obtains his critical values as predictions from response surface regressions but he does not report the regression coefficients.

compares these response surfaces to ours for case (iii) and three different lag orders at a size of 5%. For the test without lag augmentation, $q = 1$, our response surface and the ones from MacKinnon (2010) and Ericsson and MacKinnon (2002) are visually indistinguishable and they all fit nicely through the quantiles from the simulated distributions.[26]

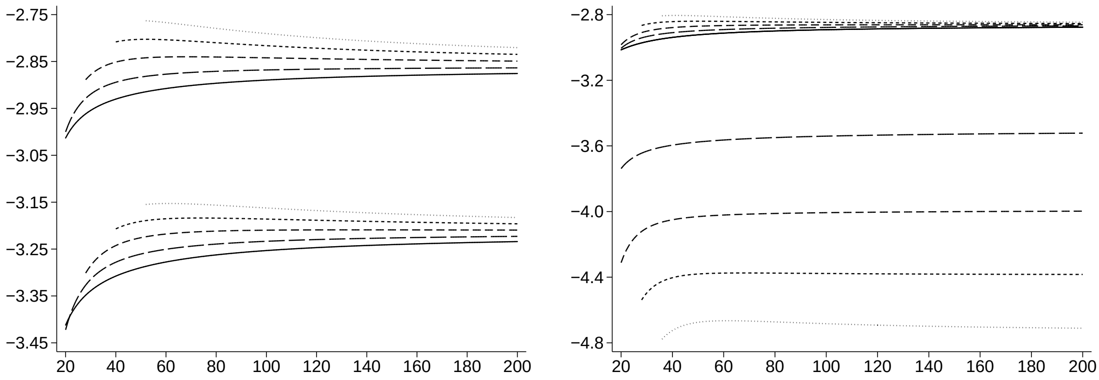
The advantage of our approach becomes apparent when we move to higher lag orders. Because the response surface from MacKinnon (2010) does not accommodate the lag augmentation, it becomes too conservative. In fact, for higher lag orders the asymptotic critical value would provide a better approximation for most sample sizes than the MacKinnon (2010) surface or the tabulated critical values from Dickey (1976). In contrast, Figure 2 reveals that our response surface provides a very good fit to the simulated critical values. It also outperforms the response surface from Cheung and Lai (1995a) that is skewed towards zero, possibly due to the smaller number of replications in their simulation and a lower polynomial order in their response surface regressions. Ericsson and MacKinnon (2002) indirectly account for the lag order by estimating response surfaces over the degrees-of-freedom adjusted sample size. However, Figure 2 clearly shows that this strategy is not appropriate for higher lag orders as the fit worsens even compared to MacKinnon (2010).

In the multivariate environment, the order of integration affects the distribution of the test statistic. Banerjee et al. (1998) and Ericsson and MacKinnon (2002) consider the t-statistic for cointegration testing under the assumption that all regressors are individually $I(1)$, the upper bound for the bounds test, but neither of them account for the lag augmentation. While the asymptotic critical value is unaffected, the response surfaces for $k = 1$ long-run forcing variable in Figure 3(a) highlight again that there are relevant differences across lag orders for small sample sizes, both for the lower and the upper bound.[27]

When we vary k for a fixed lag order $q = 1$ in Figure 3(b), the first observation is that the lower bound critical values all converge to the same asymptotic value. Pesaran et al. (2001) have previously shown that the presence of $I(0)$ regressors does not affect the asymptotic distribution of the t-statistic. There are differences for small sample sizes but they are relatively small. The picture looks different for the upper bound, where all

[26] MacKinnon (2010) is an updated version of MacKinnon (1991).

[27] The upper bound critical values are further away from zero and thus lie below their lower bound counterpart in Figure 3.



(a) $k = 1$, $q \in \{0, 3, 6, 9, 12\}$ (b) $q = 1$, $k \in \{0, 2, 4, 6, 8\}$

Figure 3: Response surfaces from equation (8) for the t-statistic in case (iii) at the 5% significance level over a range of effective sample sizes $T - \max(q, 1)$. Panel (a) shows response surfaces for selected lag orders q and fixed $k = 1$. The solid curves represent the lower bound (closer to zero) and the upper bound for $q = 0$. With increasing lag order, the curves have shorter dashes. Panel (b) shows response surfaces for selected numbers of variables k with fixed $q = 1$. The solid curve refers to $k = 0$. With increasing k, the curves have shorter dashes and are separately drawn for the lower and upper bound.

variables are purely $I(1)$. The spread between the response surfaces is largely driven by the asymptotic critical value that now depends on k. The inconclusive area between the lower and upper bound widens with increasing k as the lower bound surfaces are pulled towards zero and the upper bound surfaces are pushed into the opposite direction, relative to the solid curve that represents the response surface for $k = 0$. Similar pictures emerge for other slices through the response surface.

3.3 Combined response surface regressions

The response surface model (8) is estimated for each number of regressors k separately. This leads to an inflated number of regression results and has the additional disadvantage that critical values for large models with $k > 10$ cannot be obtained without resorting to extrapolation methods. Ericsson and MacKinnon (2002) estimate a simple meta response surface for the predicted asymptotic quantiles as a linear function of k and the number of deterministic model components. While this is useful as a crude approximation for small numbers of variables, it does not readily extend to larger models because it ignores the diminishing slope of the response surface with increasing k.

Here, we propose to introduce the number of variables as an additional predictor in the response surface regressions and to combine the simulated quantiles for all k. The separate response surfaces in Section 3.2 reveal that the marginal differences between the asymptotic quantiles become smaller with increasing k. This suggests to model the response surface with negative powers in the number of variables $1 + k$. Thus, for each triplet $\{c, d, p\}$, we consider the following regression:

$$Q(k, T, q) = \sum_{i=0}^{r} \sum_{j=0}^{m} \sum_{l=0}^{n} \theta_{i,j,l} (1 + k)^{-i} [N(T, q)]^{-j} [H(q, k)]^{l} + \nu. \tag{9}$$

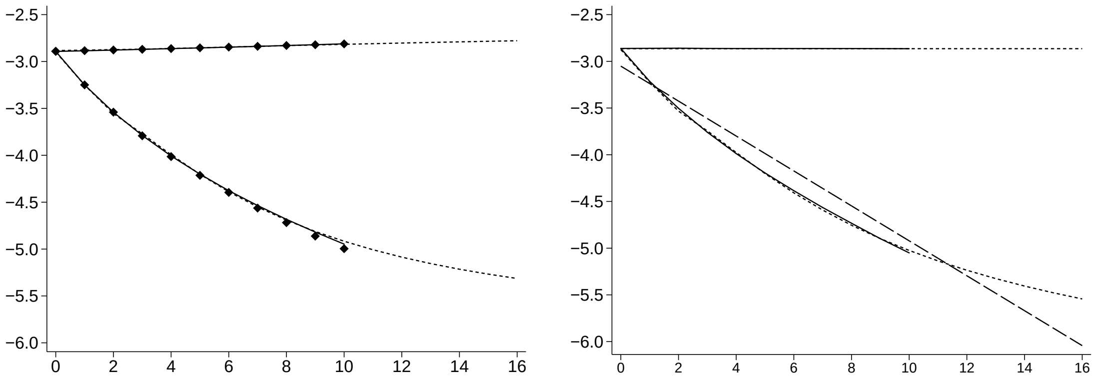
The lag order q is still uninformative for the asymptotic quantiles which implies the restrictions $\theta_{i,0,l} = 0$ for all $l > 0$. The intercept $\theta_{0,0,0}$ now has the interpretation as the asymptotic quantile when both $T \to \infty$ and $k \to \infty$. For a given k, the respective asymptotic quantile can be computed from the coefficients $\theta_{i,0,0}$. When $k = 0$, it is $\sum_{i=0}^{r} \theta_{i,0,0}$. For the t-statistic, the asymptotic distribution does not depend on k when all variables are $I(0)$. Hence, we further restrict $\theta_{i,0,0} = 0$ for all $i > 0$ in this situation.

It turns out that the orders $r = 4$, $m = 3$, and $n = 1$ yield satisfactory results. In addition, the coefficients of the interaction terms of the variable count with the inverse sample size are often statistically insignificant when the latter is raised to a higher power. We thus set $\theta_{i,j,l} = 0$ when both $i > 0$ and $j > 1$. Equation (9) thus becomes

$$\begin{aligned} Q(k, T, q) = \theta_{0,0,0} + \sum_{i=1}^{4} \theta_{i,0,0} \frac{1}{(1 + k)^{i}} + \sum_{j=1}^{3} \theta_{0,j,0} \frac{1}{[N(T, q)]^{j}} + \sum_{i=1}^{4} \theta_{i,1,0} \frac{1}{(1 + k)^{i} N(T, q)} \\ + \sum_{j=1}^{3} \theta_{0,j,1} \frac{H(q, k)}{[N(T, q)]^{j}} + \sum_{i=1}^{4} \theta_{i,1,1} \frac{H(q, k)}{(1 + k)^{i} N(T, q)} + \nu. \end{aligned} \tag{10}$$

The ordinary least squares estimates are presented in Tables 18 to 25 in Appendix B for the quantiles corresponding to a nominal size of 1%, 5%, and 10%.[28] For a given k, the fit from equation (10) is expected to be worse than from the tailored equation (8). However, Figure 4 highlights that the differences are very small and essentially negligible almost everywhere. Specification (10) allows to present the estimates in a more compact way and to compute the finite-sample critical values for any number k of long-run forcing regressors, effective sample size $N(T, q)$, and number of short-run coefficients $H(q, k)$.

[28]The coefficient estimates for other quantiles are available upon request.



(a) $T = 100$ (b) $T = \infty$

Figure 4: Response surfaces for the t-statistic in case (iii) at the 5% significance level over a range of variable numbers k. The solid curves are the response surfaces from equation (8) for the lower bound (closer to zero) and the upper bound, respectively, and the short-dashed curves from equation (10). Panel (a) shows the finite-sample results for $T = 100$ and $q = 1$. The diamonds are the critical values computed from the simulated distribution of the 10^7 t-statistics. Panel (b) shows the asymptotic results. The long-dashed line is the meta response surface from Ericsson and MacKinnon (2002) for the asymptotic upper bound critical values.

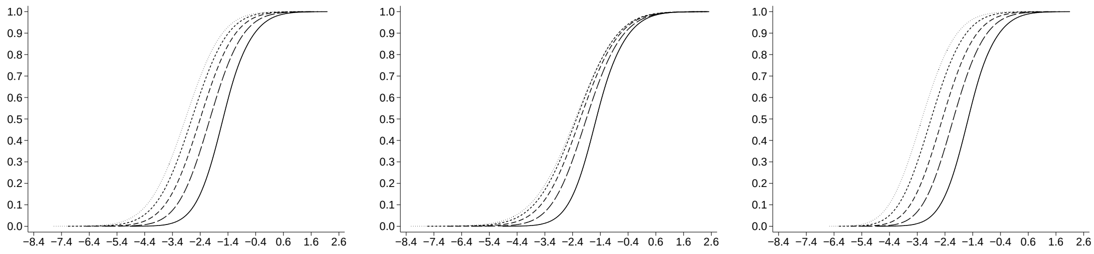
3.4 Approximate p-values

With the response surface regressions from Section 3.3 for a fine grid of quantiles, we can already describe the shape of the finite-sample and asymptotic distributions quite well. If we are interested in a p-value for a given value of the test statistic, we still need to interpolate between the two nearest quantiles. MacKinnon (1994, 1996) suggests a local approximation strategy. Consider the following regression model:

$$F^{-1}(p) = \sum_{i=0}^{n} \phi_i \left[\hat{Q}(p)\right]^i + e, \tag{11}$$

where $F^{-1}(p)$ is the inverse cumulative distribution function of the test statistic that would apply under standard asymptotics, and $\hat{Q}(p)$ is the predicted p-quantile from equation (10) for a given combination of k, T, and q.[29] If the distributional assumption was correct, then model (11) would be correctly specified with $\phi_1 = 1$ and all other coefficients being zero. $\phi_0 \neq 0$ allows for a shift in the mean and $\phi_1 \neq 1$ for a different variance. Since in our case this regression only serves as an approximation of the unknown shape of the

[29] For convenience, we are suppressing the arguments k, T, q in favor of p that is variable in this regression.



(a) $N(T,q) = 50, q = 0$ (b) $N(T,q) = 50, q = 3$ (c) $N(T,q) = \infty$

Figure 5: Implied upper-bound cumulative distribution functions from equation (10) for the t-statistic in case (iii) with $k = \{0, 2, 4, 6, 8\}$ variables for different effective sample sizes $N(T,q)$ and lag orders q. The solid curve refers to $k = 0$. With increasing k, the curves have shorter dashes.

distribution, the higher-order terms potentially help to improve the fit. It turns out that for our purpose a second-order polynomial, $n = 2$, works sufficiently well.

We follow MacKinnon (1996) and Ericsson and MacKinnon (2002) regarding the choice of 221 quantiles of the simulated distributions that we compute for both test statistics:

$$p \in \{0.0001, 0.0002, 0.0005, 0.001, \ldots, 0.01, 0.015, \\ \ldots, 0.99, 0.991, \ldots, 0.999, 0.9995, 0.9998, 0.9999\}.$$

Equation (11) is then estimated for the 9 predicted quantiles that are nearest to the observed value of the test statistic. MacKinnon (1994, 1996) notices that an OLS estimation ignores heteroskedasticity and pairwise correlation of the quantiles, and he suggests to estimate equation (11) by generalized least squares (GLS). However, we do not find that a GLS estimation uniformly improves the fit. For practical purposes, a feasible GLS estimation requires estimates of the variances of the respective quantiles. While the variance estimates can in principle be obtained from the response surface regressions, this would require to supply the variance-covariance matrices from all estimations together with the computer program that computes the approximate p-values. From our perspective, it seems worth to trade off minor efficiency gains for the convenience of not having to store this bulk of data.

Finally, the approximate p-value corresponding to the observed value of the test statistic τ is computed as

$$\hat{p} = F\left(\sum_{i=0}^{n} \hat{\phi}_i \tau^i\right), \tag{12}$$

where $\hat{\phi}_i$ are the coefficient estimates from equation (11). This procedure to approximate p-values is implemented in the *Stata* program described by Kripfganz and Schneider (2018) for both the F-statistic and the t-statistic. The necessity to compute separate critical values and p-values for finite samples and to account for the number of short-run coefficients becomes apparent again in Figure 5 that displays a selection of the implied cumulative distribution functions for the t-statistic. It is clearly visible that the left tail of the distribution is more spread out in finite samples compared to the asymptotic distribution, and that higher lag orders can have a substantial effect on the shape of the finite-sample distribution.

4 Conclusion

The Pesaran et al. (2001) bounds test for the existence of a level relationship is widely applied in the empirical practice. The current paper provides response surface estimates for the respective lower and upper bound critical values, corresponding to the situations where all long-run forcing variables are either $I(0)$ or $I(1)$, respectively. Precise finite-sample and asymptotic critical values for various cases of unrestricted or restricted deterministic model components and any number of long-run forcing variables can be computed directly from the regression tables. While such critical values have been reported previously in the literature, they often only cover a rather small subset of the possibility space and are typically less precise due to a smaller number of replications in the respective Monte Carlo simulations.

With the exception of Cheung and Lai (1995a) for the augmented Dickey-Fuller test that results as a special case of the framework considered here, the previously obtained response surfaces do not account for the lag augmentation in the underlying regression model. With our response surface estimates, accurate finite-sample critical value bounds can be obtained for any number of short-run coefficients. In practice, the correct lag order is usually unknown and possibly different across variables. For the purpose of efficient estimation of the model coefficients, the optimal lag order is often obtained with model selection criteria such as the Akaike or Schwarz information criterion. However, as stressed by Pesaran et al. (2001), for testing purposes it is of primary concern that the error term is free of serial correlation. As long as there are enough degrees of freedom available, additional lags of the variables can help to achieve this aim. Once a conclusion from the test is drawn, a more parsimonious model can be estimated along the lines of the Pesaran and Shin (1998) autoregressive distributed lag (ARDL) modelling approach. In the statistical software *Stata*, the ARDL and EC models can be estimated with the program provided by Kripfganz and Schneider (2018). Our response surface estimates and the procedure to obtain approximate p-values are incorporated in this program.[30]

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[30] The program can be installed from `http://www.kripfganz.de/stata/`.

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Appendix A Details on the computational methods

All computations are performed in *Stata* 15. The bulk of computations, the simulations, are performed in *Stata*'s integrated matrix language, *Mata*. As a byte-compiled language, *Mata* runs about 5 to 6 times slower than a high-performance, compiled language such as *C*. However, most *Mata* functions used in our simulations hook directly into compiled ones, such as *LAPACK* functions (Anderson et al., 1999), which decreases the speed disadvantage substantially. A reasonable and conservative presumption for our simulation is that we run about half as fast as pure *C* would. *Mata*, however, is much more user friendly than *C*. For example, an appropriate random number generation mechanism that has a sufficiently large period and that accommodates parallel computations is readily available. For that, we use random number streams based on the Mersenne Twister pseudorandom number generator. Overall, we believe that *Mata* provides a good balance between speed and high-level language features. We run our computations in parallel on 35 cores, each of which running at 2.9 GHz. After the removal of any redundant calculations, such as repeated calculation of the same cross products, the simulations conclude after about three days.

Storing the calculated statistics is a desirable computational aspect of the simulation. It has the critical advantage that it isolates sequential steps that are computationally intensive. Once the statistics are saved, any subsequent operations can be done independently, without re-calculating the results from the previous step over and over again, should either bugs or additional research ideas pop up. However, the large number of calculated statistics, roughly 100 billion F-statistics and 60 billion t-statistics, poses several problems, the most serious one being storage. Using floating point numbers with 8 digit precision (4 bytes per number), the (uncompressed) storage required is 640 GB. While this is not technically infeasible, it is too much of a hindrance for practical research. Our solution was to round the calculated statistics to three digits after the decimal point. We then further transformed the rounded numbers in terms of first differences of sorted statistics and occurrence counts. The transformation is completely reversible, so that the original rounded 10 billion statistics per simulation design can be fully recovered. The resulting storage requirements are 40 GB, which decrease further to 8 GB when adding a conventional compression algorithm. This magnitude is easily manageable.

Appendix B Tables

Table 2: Response surface estimates, F-statistic, lower bound, case (i)

k	α	$\theta_{0,0}$	$\theta_{1,0}$	$\theta_{2,0}$	$\theta_{3,0}$	$\theta_{1,1}$	$\theta_{2,1}$	$\theta_{3,1}$	SE($\theta_{0,0}$)	$\bar{R}^2$	RMSE
0	1%	6.8875	29.187	-48.36	706.4	-0.750	-0.47	813.1	0.0017	0.986	0.091
	5%	4.1053	12.271	-94.58	958.5	-0.627	23.14	-31.7	0.0006	0.983	0.033
	10%	2.9626	7.471	-91.34	862.4	-0.431	20.85	-111.8	0.0004	0.978	0.021
1	1%	4.7135	22.668	34.02	839.3	-0.302	-11.11	1135.8	0.0011	0.990	0.062
	5%	3.1042	9.650	-0.94	317.0	-0.288	1.26	313.1	0.0004	0.988	0.025
	10%	2.4078	5.744	-9.46	226.0	-0.237	2.52	144.3	0.0003	0.983	0.017
2	1%	3.8491	21.822	-47.64	3414.6	-0.619	13.15	789.1	0.0009	0.994	0.047
	5%	2.6738	9.348	-4.25	1092.2	-0.470	11.95	142.4	0.0004	0.992	0.021
	10%	2.1503	5.641	-0.33	577.7	-0.382	8.95	34.5	0.0002	0.988	0.014
3	1%	3.3558	23.794	-132.74	5112.0	-0.491	-10.91	1419.0	0.0007	0.996	0.035
	5%	2.4140	11.339	-62.13	1957.7	-0.400	-4.75	581.3	0.0004	0.994	0.018
	10%	1.9873	7.284	-44.03	1176.8	-0.341	-3.53	362.5	0.0002	0.991	0.013
4	1%	3.0386	24.475	-149.53	5806.6	-0.395	-34.16	2302.0	0.0007	0.997	0.030
	5%	2.2442	11.792	-51.95	1943.6	-0.377	-15.14	1010.2	0.0004	0.995	0.016
	10%	1.8787	7.765	-35.93	1088.3	-0.338	-10.74	664.7	0.0003	0.992	0.013
5	1%	2.8243	23.079	-101.35	6237.5	-0.519	-24.09	2469.0	0.0006	0.997	0.025
	5%	2.1241	12.010	-62.03	2502.8	-0.430	-13.81	1247.4	0.0003	0.996	0.015
	10%	1.7999	8.228	-54.36	1575.3	-0.377	-11.00	881.2	0.0003	0.993	0.012
6	1%	2.6584	23.357	-163.53	8573.9	-0.566	-19.04	2819.3	0.0006	0.998	0.021
	5%	2.0323	12.195	-85.76	3484.8	-0.473	-9.34	1436.7	0.0003	0.997	0.012
	10%	1.7397	8.387	-69.49	2189.8	-0.418	-6.99	1015.9	0.0002	0.996	0.010
7	1%	2.5221	25.664	-321.24	12416.1	-0.538	-27.69	3450.4	0.0007	0.998	0.022
	5%	1.9556	13.404	-150.84	5045.0	-0.459	-14.47	1814.9	0.0004	0.997	0.013
	10%	1.6885	9.300	-114.41	3206.4	-0.410	-11.00	1310.9	0.0003	0.996	0.010
8	1%	2.4126	28.117	-527.16	17583.6	-0.563	-27.01	3942.8	0.0006	0.999	0.021
	5%	1.8947	14.143	-209.19	6797.3	-0.478	-13.14	2097.6	0.0003	0.998	0.012
	10%	1.6481	9.658	-141.47	4161.9	-0.428	-9.50	1516.9	0.0003	0.997	0.010
9	1%	2.3251	28.065	-556.19	20221.0	-0.550	-28.71	4369.8	0.0007	0.998	0.020
	5%	1.8434	14.658	-244.19	8265.1	-0.473	-14.78	2391.1	0.0004	0.997	0.012
	10%	1.6130	10.262	-177.01	5271.3	-0.426	-11.19	1768.8	0.0003	0.996	0.010
10	1%	2.2538	27.294	-558.99	22795.4	-0.556	-30.81	5322.2	0.0006	0.998	0.017
	5%	1.8014	14.493	-256.90	9683.7	-0.474	-18.00	3180.1	0.0003	0.998	0.011
	10%	1.5848	10.022	-178.29	6106.3	-0.429	-13.61	2425.4	0.0003	0.996	0.009

Note: The response surface regression model is equation (8). The dependent variable is the simulated α-quantile of the test statistic. Separate regressions are run for each number k of weakly exogenous regressors $\mathbf{x}_t$ in equation (6). SE($\theta_{0,0}$) denotes the heteroskedasticity-robust standard error of the intercept, $\bar{R}^2$ the adjusted coefficient of determination, and RMSE the root mean square error.

Table 3: Response surface estimates, F-statistic, upper bound, case (i)

k	α	$\theta_{0,0}$	$\theta_{1,0}$	$\theta_{2,0}$	$\theta_{3,0}$	$\theta_{1,1}$	$\theta_{2,1}$	$\theta_{3,1}$	SE($\theta_{0,0}$)	$\bar{R}^2$	RMSE
0	1%	6.8875	29.187	-48.36	706.4	-0.750	-0.47	813.1	0.0017	0.986	0.091
	5%	4.1053	12.271	-94.58	958.5	-0.627	23.14	-31.7	0.0006	0.983	0.033
	10%	2.9626	7.471	-91.34	862.4	-0.431	20.85	-111.8	0.0004	0.978	0.021
1	1%	5.8446	27.970	39.91	878.2	-0.073	-5.16	1518.2	0.0012	0.993	0.069
	5%	4.0493	11.417	5.45	320.8	-0.163	7.21	390.9	0.0005	0.993	0.025
	10%	3.2454	6.503	-7.83	272.0	-0.128	6.45	171.3	0.0003	0.992	0.016
2	1%	5.1368	31.304	-117.99	4609.6	0.008	12.82	1609.5	0.0012	0.995	0.061
	5%	3.7851	13.214	-24.12	1403.0	-0.086	14.90	417.1	0.0005	0.996	0.023
	10%	3.1598	7.680	-11.09	726.4	-0.075	10.86	186.3	0.0003	0.995	0.014
3	1%	4.7040	29.674	-6.19	5284.2	-0.063	36.43	1218.7	0.0008	0.997	0.043
	5%	3.5887	13.409	25.28	1560.5	-0.061	21.23	348.3	0.0004	0.997	0.017
	10%	3.0652	8.165	16.41	827.4	-0.049	14.00	166.4	0.0002	0.997	0.011
4	1%	4.3928	31.836	-64.25	7647.1	0.083	18.61	1967.8	0.0010	0.998	0.039
	5%	3.4371	14.924	7.57	2437.9	0.007	14.90	651.6	0.0004	0.998	0.016
	10%	2.9832	9.189	11.98	1278.7	-0.006	10.75	339.8	0.0003	0.998	0.010
5	1%	4.1779	29.702	86.47	7701.6	-0.045	41.15	1616.5	0.0008	0.998	0.032
	5%	3.3265	14.408	81.75	2306.6	-0.041	24.65	524.6	0.0003	0.998	0.014
	10%	2.9192	9.106	55.87	1216.7	-0.030	16.36	281.1	0.0002	0.998	0.009
6	1%	3.9941	32.939	-33.92	11544.7	0.062	24.65	2267.8	0.0007	0.999	0.028
	5%	3.2303	16.095	42.64	3757.3	0.015	17.37	821.6	0.0003	0.999	0.012
	10%	2.8616	10.233	38.51	1997.2	0.003	12.49	450.9	0.0002	0.999	0.008
7	1%	3.8503	35.919	-212.97	17139.0	0.042	26.35	2565.3	0.0008	0.999	0.027
	5%	3.1535	17.427	-3.04	5639.3	0.009	18.55	942.1	0.0003	0.999	0.011
	10%	2.8143	11.150	15.73	3065.6	0.001	13.38	528.9	0.0002	0.999	0.007
8	1%	3.7253	41.566	-573.99	26110.7	0.045	20.27	3137.6	0.0009	0.999	0.029
	5%	3.0868	19.641	-113.62	8761.4	0.010	17.45	1157.4	0.0003	0.999	0.011
	10%	2.7728	12.491	-39.84	4781.1	0.001	13.15	650.2	0.0002	0.999	0.007
9	1%	3.6300	41.281	-562.94	29869.2	0.061	19.85	3545.5	0.0009	0.999	0.027
	5%	3.0327	19.932	-108.85	10359.2	0.019	17.32	1364.9	0.0004	0.999	0.011
	10%	2.7379	12.842	-36.82	5757.2	0.009	12.79	795.7	0.0002	0.999	0.007
10	1%	3.5472	42.192	-587.36	33958.3	0.065	18.73	3494.3	0.0008	0.999	0.022
	5%	2.9848	21.090	-146.81	12623.0	0.021	16.97	1320.1	0.0003	0.999	0.009
	10%	2.7069	13.660	-56.81	7099.3	0.011	12.79	762.8	0.0002	0.999	0.006

Note: The response surface regression model is equation (8). The dependent variable is the simulated α-quantile of the test statistic. Separate regressions are run for each number k of weakly exogenous regressors $\mathbf{x}_t$ in equation (6). SE($\theta_{0,0}$) denotes the heteroskedasticity-robust standard error of the intercept, $\bar{R}^2$ the adjusted coefficient of determination, and RMSE the root mean square error.

Table 4: Response surface estimates, F-statistic, lower bound, case (ii)

k	α	$\theta_{0,0}$	$\theta_{1,0}$	$\theta_{2,0}$	$\theta_{3,0}$	$\theta_{1,1}$	$\theta_{2,1}$	$\theta_{3,1}$	SE($\theta_{0,0}$)	$\bar{R}^2$	RMSE
0	1%	6.3769	28.932	226.25	-1520.9	-0.906	-60.16	2021.6	0.0012	0.993	0.075
	5%	4.5831	12.625	79.12	-667.0	-1.117	-9.37	574.0	0.0005	0.990	0.032
	10%	3.7792	7.444	42.51	-408.1	-0.987	-2.23	284.6	0.0003	0.984	0.021
1	1%	4.8785	26.267	69.67	2336.8	-1.172	9.45	1078.3	0.0009	0.995	0.050
	5%	3.5974	11.744	38.59	662.4	-0.989	11.09	279.8	0.0004	0.995	0.020
	10%	3.0150	7.141	19.81	387.0	-0.869	9.11	114.0	0.0002	0.994	0.013
2	1%	4.0934	26.566	-31.44	4698.6	-0.956	-3.40	1452.7	0.0008	0.997	0.040
	5%	3.0836	12.090	10.41	1514.4	-0.863	5.80	425.2	0.0003	0.997	0.016
	10%	2.6175	7.422	12.95	778.1	-0.767	4.90	214.2	0.0002	0.996	0.011
3	1%	3.6031	28.947	-198.99	7953.9	-0.816	-19.97	2017.6	0.0008	0.998	0.033
	5%	2.7620	13.307	-38.61	2488.8	-0.748	-5.75	782.8	0.0003	0.998	0.014
	10%	2.3688	8.408	-14.23	1282.1	-0.678	-3.86	480.8	0.0002	0.997	0.010
4	1%	3.2778	26.433	-85.00	7477.1	-0.738	-25.03	2407.6	0.0007	0.998	0.026
	5%	2.5448	12.919	-6.99	2460.0	-0.690	-9.88	1047.0	0.0003	0.998	0.013
	10%	2.2001	8.479	-1.63	1320.5	-0.635	-7.08	684.2	0.0002	0.997	0.009
5	1%	3.0379	27.168	-165.03	9944.4	-0.759	-29.36	2963.4	0.0006	0.999	0.022
	5%	2.3851	13.620	-47.51	3562.1	-0.694	-11.30	1338.2	0.0003	0.999	0.011
	10%	2.0766	9.082	-32.18	2074.7	-0.635	-7.74	898.4	0.0002	0.998	0.009
6	1%	2.8515	28.119	-284.10	13560.4	-0.774	-22.92	3199.6	0.0006	0.999	0.021
	5%	2.2611	14.333	-100.91	5084.5	-0.677	-11.30	1598.1	0.0003	0.999	0.010
	10%	1.9806	9.670	-68.75	3054.5	-0.618	-8.54	1124.7	0.0002	0.998	0.008
7	1%	2.6974	32.043	-578.21	20047.3	-0.743	-35.30	4044.9	0.0007	0.999	0.022
	5%	2.1606	16.000	-208.71	7526.5	-0.649	-18.00	2071.9	0.0003	0.999	0.011
	10%	1.9028	10.861	-137.52	4553.9	-0.591	-14.14	1508.2	0.0002	0.998	0.009
8	1%	2.5798	31.021	-580.72	22624.3	-0.735	-29.14	4298.2	0.0006	0.999	0.019
	5%	2.0812	15.912	-223.09	8837.0	-0.641	-15.72	2320.5	0.0003	0.999	0.010
	10%	1.8412	10.958	-152.31	5459.2	-0.585	-12.87	1731.7	0.0002	0.998	0.008
9	1%	2.4826	29.683	-551.60	24821.1	-0.722	-27.61	4987.9	0.0006	0.999	0.017
	5%	2.0159	15.438	-219.10	10044.9	-0.632	-15.91	2927.5	0.0003	0.999	0.009
	10%	1.7905	10.708	-154.09	6335.3	-0.576	-13.61	2284.5	0.0002	0.998	0.007
10	1%	2.3987	29.370	-543.71	26930.0	-0.702	-30.75	5412.1	0.0006	0.999	0.016
	5%	1.9586	15.912	-249.88	11602.6	-0.617	-17.75	3179.3	0.0003	0.998	0.010
	10%	1.7458	11.254	-188.26	7592.3	-0.565	-14.73	2458.4	0.0003	0.998	0.008

Note: The response surface regression model is equation (8). The dependent variable is the simulated α-quantile of the test statistic. Separate regressions are run for each number k of weakly exogenous regressors $\mathbf{x}_t$ in equation (6). SE($\theta_{0,0}$) denotes the heteroskedasticity-robust standard error of the intercept, $\bar{R}^2$ the adjusted coefficient of determination, and RMSE the root mean square error.

Table 5: Response surface estimates, F-statistic, upper bound, case (ii)

k	α	$\theta_{0,0}$	$\theta_{1,0}$	$\theta_{2,0}$	$\theta_{3,0}$	$\theta_{1,1}$	$\theta_{2,1}$	$\theta_{3,1}$	SE($\theta_{0,0}$)	$\bar{R}^2$	RMSE
0	1%	6.3769	28.932	226.25	-1520.9	-0.906	-60.16	2021.6	0.0012	0.993	0.075
	5%	4.5831	12.625	79.12	-667.0	-1.117	-9.37	574.0	0.0005	0.990	0.032
	10%	3.7792	7.444	42.51	-408.1	-0.987	-2.23	284.6	0.0003	0.984	0.021
1	1%	5.4618	32.320	43.15	2824.7	-0.345	10.61	1590.0	0.0009	0.997	0.053
	5%	4.1084	15.078	19.97	915.9	-0.369	14.37	450.7	0.0004	0.997	0.020
	10%	3.4855	9.450	6.20	536.4	-0.324	11.02	215.4	0.0002	0.997	0.013
2	1%	4.9199	34.587	-40.57	5283.9	0.073	-1.69	2360.2	0.0011	0.997	0.052
	5%	3.8155	16.397	11.69	1555.0	-0.088	9.06	788.2	0.0004	0.998	0.020
	10%	3.2969	10.430	8.63	808.8	-0.101	7.50	431.8	0.0003	0.998	0.012
3	1%	4.5632	37.496	-222.89	10249.9	-0.047	25.14	2073.5	0.0010	0.998	0.042
	5%	3.6167	17.470	-20.18	3027.9	-0.086	20.55	665.5	0.0004	0.999	0.016
	10%	3.1663	11.076	2.00	1538.3	-0.085	15.45	342.9	0.0002	0.999	0.010
4	1%	4.3109	35.073	-60.22	10305.1	0.061	23.68	2355.8	0.0009	0.998	0.037
	5%	3.4712	17.189	37.78	3212.4	-0.029	21.23	773.1	0.0004	0.999	0.015
	10%	3.0679	11.196	33.82	1685.9	-0.040	15.86	414.8	0.0002	0.998	0.010
5	1%	4.1121	37.352	-142.31	13805.1	0.089	12.55	2922.8	0.0008	0.999	0.031
	5%	3.3551	18.606	10.03	4568.0	0.005	14.96	1063.2	0.0003	0.999	0.013
	10%	2.9892	12.159	22.13	2444.9	-0.014	12.46	582.7	0.0002	0.999	0.008
6	1%	3.9571	38.172	-215.56	18262.4	0.004	35.42	2538.4	0.0008	0.999	0.029
	5%	3.2641	18.945	9.01	5997.1	-0.022	25.23	911.7	0.0003	0.999	0.012
	10%	2.9266	12.474	26.64	3249.7	-0.026	18.71	492.5	0.0002	0.999	0.008
7	1%	3.8218	44.015	-604.70	27864.0	-0.019	29.05	3169.8	0.0010	0.999	0.031
	5%	3.1856	21.334	-117.79	9467.2	-0.033	23.84	1144.8	0.0004	0.999	0.012
	10%	2.8730	13.876	-35.90	5127.8	-0.030	18.20	624.9	0.0002	0.999	0.008
8	1%	3.7172	43.106	-546.56	31034.3	-0.008	37.79	2981.2	0.0008	0.999	0.027
	5%	3.1216	21.499	-105.36	11043.5	-0.022	27.46	1091.5	0.0003	0.999	0.011
	10%	2.8282	14.157	-28.12	6078.9	-0.022	20.33	609.9	0.0002	0.999	0.007
9	1%	3.6248	43.962	-570.88	35329.7	0.031	35.17	2827.7	0.0008	0.999	0.023
	5%	3.0651	22.375	-123.66	13046.4	-0.001	26.51	1004.8	0.0003	0.999	0.009
	10%	2.7884	14.939	-44.40	7392.6	-0.007	19.54	561.9	0.0002	0.999	0.006
10	1%	3.5518	41.779	-364.97	35425.4	0.040	31.98	3421.2	0.0008	0.999	0.019
	5%	3.0187	22.194	-68.96	13851.5	0.008	23.85	1357.8	0.0003	0.999	0.008
	10%	2.7554	15.028	-17.25	8047.5	0.000	17.86	795.0	0.0002	0.999	0.006

Note: The response surface regression model is equation (8). The dependent variable is the simulated α-quantile of the test statistic. Separate regressions are run for each number k of weakly exogenous regressors $\mathbf{x}_t$ in equation (6). SE($\theta_{0,0}$) denotes the heteroskedasticity-robust standard error of the intercept, $\bar{R}^2$ the adjusted coefficient of determination, and RMSE the root mean square error.

Table 6: Response surface estimates, F-statistic, lower bound, case (iii)

k	α	$\theta_{0,0}$	$\theta_{1,0}$	$\theta_{2,0}$	$\theta_{3,0}$	$\theta_{1,1}$	$\theta_{2,1}$	$\theta_{3,1}$	SE($\theta_{0,0}$)	$\bar{R}^2$	RMSE
0	1%	11.7570	43.861	306.37	-2861.6	-4.037	-28.71	2832.9	0.0024	0.981	0.179
	5%	8.1893	16.491	77.09	-997.0	-3.686	27.36	580.0	0.0011	0.946	0.091
	10%	6.5903	8.444	21.87	-414.0	-3.113	26.22	182.6	0.0008	0.893	0.064
1	1%	6.8187	33.223	-28.85	4086.1	-2.015	42.84	993.4	0.0012	0.993	0.071
	5%	4.9055	13.345	-14.20	1463.9	-1.586	31.21	115.3	0.0005	0.989	0.033
	10%	4.0346	7.442	-24.28	997.6	-1.356	23.73	-31.1	0.0004	0.980	0.023
2	1%	5.1280	29.192	-16.16	4569.6	-1.136	-2.71	1783.4	0.0012	0.995	0.053
	5%	3.7841	12.223	17.43	1344.5	-1.009	7.63	521.5	0.0005	0.994	0.022
	10%	3.1638	6.808	22.06	578.7	-0.876	5.98	266.2	0.0003	0.992	0.015
3	1%	4.2658	29.088	-145.33	7753.9	-1.030	0.92	1706.0	0.0009	0.997	0.037
	5%	3.2112	12.389	-9.60	2302.1	-0.879	7.13	537.1	0.0003	0.997	0.015
	10%	2.7190	7.261	6.75	1141.8	-0.770	5.96	272.7	0.0002	0.997	0.010
4	1%	3.7410	26.457	-53.15	7531.7	-0.865	-8.31	2081.1	0.0007	0.998	0.029
	5%	2.8601	12.012	18.12	2328.2	-0.760	0.60	789.7	0.0003	0.998	0.012
	10%	2.4460	7.374	20.32	1177.7	-0.679	1.21	458.4	0.0002	0.998	0.008
5	1%	3.3828	27.150	-138.63	10004.3	-0.826	-17.91	2652.7	0.0006	0.998	0.024
	5%	2.6202	12.815	-22.08	3380.4	-0.720	-4.22	1091.7	0.0003	0.999	0.010
	10%	2.2599	8.123	-9.45	1878.0	-0.644	-2.10	683.5	0.0002	0.998	0.007
6	1%	3.1213	27.473	-228.84	13186.1	-0.818	-11.21	2791.9	0.0006	0.999	0.022
	5%	2.4453	13.348	-64.85	4740.2	-0.694	-3.27	1274.8	0.0003	0.999	0.009
	10%	2.1240	8.645	-38.20	2733.0	-0.619	-2.31	851.4	0.0002	0.999	0.007
7	1%	2.9146	31.561	-540.36	19890.5	-0.778	-24.28	3625.6	0.0007	0.999	0.022
	5%	2.3094	15.034	-172.63	7159.2	-0.660	-10.09	1724.9	0.0003	0.999	0.010
	10%	2.0189	9.801	-103.30	4172.4	-0.588	-7.54	1198.4	0.0002	0.999	0.007
8	1%	2.7598	30.262	-526.09	22151.7	-0.760	-17.91	3778.4	0.0006	0.999	0.019
	5%	2.2043	15.008	-187.17	8435.3	-0.642	-8.63	1946.6	0.0003	0.999	0.009
	10%	1.9372	9.981	-118.54	5049.5	-0.576	-6.84	1394.1	0.0002	0.999	0.007
9	1%	2.6331	29.426	-528.97	24762.7	-0.732	-19.06	4428.0	0.0006	0.999	0.017
	5%	2.1194	14.693	-186.17	9598.8	-0.626	-9.52	2451.1	0.0003	0.999	0.008
	10%	1.8712	9.926	-125.21	5929.0	-0.563	-8.12	1856.4	0.0002	0.999	0.006
10	1%	2.5287	28.641	-487.15	26207.1	-0.710	-22.09	4869.5	0.0006	0.999	0.015
	5%	2.0475	15.161	-215.58	11107.1	-0.608	-12.01	2746.5	0.0003	0.999	0.009
	10%	1.8152	10.448	-155.76	7105.5	-0.550	-9.62	2060.5	0.0002	0.998	0.007

Note: The response surface regression model is equation (8). The dependent variable is the simulated α-quantile of the test statistic. Separate regressions are run for each number k of weakly exogenous regressors $\mathbf{x}_t$ in equation (6). SE($\theta_{0,0}$) denotes the heteroskedasticity-robust standard error of the intercept, $\bar{R}^2$ the adjusted coefficient of determination, and RMSE the root mean square error.

Table 7: Response surface estimates, F-statistic, upper bound, case (iii)

k	α	$\theta_{0,0}$	$\theta_{1,0}$	$\theta_{2,0}$	$\theta_{3,0}$	$\theta_{1,1}$	$\theta_{2,1}$	$\theta_{3,1}$	SE($\theta_{0,0}$)	$\bar{R}^2$	RMSE
0	1%	11.7570	43.861	306.37	-2861.6	-4.037	-28.71	2832.9	0.0024	0.981	0.179
	5%	8.1893	16.491	77.09	-997.0	-3.686	27.36	580.0	0.0011	0.946	0.091
	10%	6.5903	8.444	21.87	-414.0	-3.113	26.22	182.6	0.0008	0.893	0.064
1	1%	7.7358	41.914	-47.35	4635.9	-0.976	41.28	1598.1	0.0013	0.996	0.076
	5%	5.7040	18.262	-42.30	1849.9	-0.862	31.91	316.5	0.0006	0.994	0.035
	10%	4.7675	10.770	-48.49	1270.0	-0.755	23.10	88.0	0.0004	0.988	0.026
2	1%	6.2655	40.712	-65.42	5780.3	0.003	-4.17	2856.5	0.0014	0.997	0.065
	5%	4.7894	18.205	-1.16	1604.2	-0.174	6.49	984.5	0.0006	0.997	0.025
	10%	4.0949	10.958	-2.87	807.4	-0.194	4.77	556.8	0.0004	0.996	0.017
3	1%	5.4927	41.005	-219.01	10665.3	-0.153	35.87	2145.9	0.0011	0.998	0.049
	5%	4.3026	18.221	-15.94	2996.3	-0.163	24.37	679.9	0.0004	0.998	0.019
	10%	3.7360	10.956	4.11	1462.0	-0.160	17.57	341.6	0.0003	0.998	0.012
4	1%	5.0052	37.501	-49.78	10659.6	-0.001	32.20	2400.4	0.0010	0.998	0.042
	5%	3.9917	17.658	35.61	3312.6	-0.088	25.69	744.7	0.0004	0.998	0.017
	10%	3.5052	10.932	32.72	1695.6	-0.095	18.64	379.2	0.0003	0.998	0.011
5	1%	4.6578	39.464	-145.06	14334.9	0.056	17.84	2973.2	0.0009	0.999	0.034
	5%	3.7704	18.852	8.38	4670.2	-0.034	18.36	1027.3	0.0004	0.999	0.014
	10%	3.3408	11.843	19.01	2484.1	-0.048	14.30	550.0	0.0003	0.999	0.009
6	1%	4.4025	39.981	-225.90	18881.6	-0.016	39.12	2602.0	0.0009	0.999	0.032
	5%	3.6071	19.078	5.16	6137.1	-0.046	27.02	899.1	0.0004	0.999	0.013
	10%	3.2189	12.170	19.32	3340.5	-0.048	19.25	482.8	0.0002	0.999	0.008
7	1%	4.1964	45.367	-608.38	28419.2	-0.045	34.42	3135.7	0.0010	0.999	0.033
	5%	3.4764	21.309	-120.67	9593.9	-0.055	26.25	1096.2	0.0004	0.999	0.013
	10%	3.1222	13.450	-40.61	5195.8	-0.049	19.23	592.9	0.0003	0.999	0.008
8	1%	4.0382	44.180	-549.34	31594.4	-0.020	40.53	3012.3	0.0009	0.999	0.028
	5%	3.3727	21.403	-107.32	11133.2	-0.036	28.68	1071.3	0.0004	0.999	0.011
	10%	3.0445	13.787	-39.31	6245.6	-0.035	20.37	601.5	0.0002	0.999	0.008
9	1%	3.9048	44.723	-568.27	35781.7	0.018	38.66	2789.0	0.0009	0.999	0.024
	5%	3.2853	22.324	-133.27	13251.1	-0.011	26.93	1005.5	0.0004	0.999	0.010
	10%	2.9792	14.447	-50.31	7473.5	-0.018	19.83	536.9	0.0002	0.999	0.007
10	1%	3.7986	42.582	-375.69	36128.4	0.035	33.27	3466.9	0.0008	0.999	0.020
	5%	3.2145	22.051	-78.77	14078.1	-0.002	24.30	1348.6	0.0003	0.999	0.009
	10%	2.9257	14.493	-23.03	8135.8	-0.009	17.91	778.8	0.0002	0.999	0.006

Note: The response surface regression model is equation (8). The dependent variable is the simulated α-quantile of the test statistic. Separate regressions are run for each number k of weakly exogenous regressors $\mathbf{x}_t$ in equation (6). SE($\theta_{0,0}$) denotes the heteroskedasticity-robust standard error of the intercept, $\bar{R}^2$ the adjusted coefficient of determination, and RMSE the root mean square error.

Table 8: Response surface estimates, F-statistic, lower bound, case (iv)

k	α	$\theta_{0,0}$	$\theta_{1,0}$	$\theta_{2,0}$	$\theta_{3,0}$	$\theta_{1,1}$	$\theta_{2,1}$	$\theta_{3,1}$	SE($\theta_{0,0}$)	$\bar{R}^2$	RMSE
0	1%	8.2726	45.413	154.08	1124.1	-2.656	4.29	2511.0	0.0017	0.991	0.136
	5%	6.2605	21.046	65.64	69.9	-2.254	31.51	597.1	0.0008	0.985	0.070
	10%	5.3366	13.098	36.97	-82.9	-1.847	25.94	258.5	0.0006	0.975	0.052
1	1%	6.0697	40.449	-180.48	8097.0	-1.812	42.40	1486.2	0.0013	0.995	0.070
	5%	4.6674	18.888	-72.19	3074.4	-1.501	34.71	312.5	0.0006	0.994	0.035
	10%	4.0162	11.854	-52.40	1862.5	-1.296	25.80	108.9	0.0004	0.990	0.026
2	1%	4.9692	31.697	51.30	6576.4	-1.337	20.93	1756.0	0.0010	0.996	0.056
	5%	3.8699	14.547	82.80	1696.1	-1.169	19.76	493.0	0.0005	0.995	0.026
	10%	3.3556	8.860	71.23	669.4	-1.053	15.83	212.0	0.0004	0.994	0.019
3	1%	4.2898	32.093	-71.62	9641.5	-1.183	2.06	2119.0	0.0008	0.998	0.036
	5%	3.3804	14.797	44.85	2841.1	-1.059	10.87	654.3	0.0003	0.998	0.016
	10%	2.9508	9.222	47.72	1375.5	-0.963	9.88	314.9	0.0002	0.997	0.012
4	1%	3.8394	28.233	48.59	9788.0	-1.121	18.58	1581.3	0.0007	0.998	0.028
	5%	3.0509	13.805	80.96	3052.3	-0.967	13.02	567.5	0.0003	0.999	0.013
	10%	2.6780	8.785	71.64	1461.7	-0.878	9.55	306.0	0.0002	0.998	0.009
5	1%	3.5043	31.201	-197.13	15048.0	-1.052	-1.08	2486.7	0.0007	0.999	0.025
	5%	2.8099	15.343	-16.90	5094.3	-0.902	1.65	1069.2	0.0003	0.999	0.010
	10%	2.4793	9.939	8.12	2741.5	-0.825	1.94	647.6	0.0002	0.999	0.007
6	1%	3.2440	34.974	-518.47	22224.5	-0.956	-13.29	3169.2	0.0008	0.999	0.025
	5%	2.6257	16.655	-119.17	7578.0	-0.839	-3.69	1412.4	0.0003	0.999	0.010
	10%	2.3278	10.885	-54.94	4230.4	-0.773	-2.16	918.3	0.0002	0.999	0.007
7	1%	3.0511	33.952	-524.65	24792.2	-0.936	-16.70	3659.6	0.0006	0.999	0.020
	5%	2.4844	16.891	-154.27	9160.4	-0.818	-6.23	1746.1	0.0003	0.999	0.009
	10%	2.2108	11.225	-82.09	5273.1	-0.751	-4.14	1172.9	0.0002	0.999	0.006
8	1%	2.8925	33.156	-544.74	27841.4	-0.895	-13.89	3907.1	0.0006	0.999	0.018
	5%	2.3687	17.001	-178.26	10697.4	-0.781	-6.90	1987.5	0.0003	0.999	0.008
	10%	2.1153	11.515	-107.24	6391.1	-0.719	-5.24	1384.4	0.0002	0.999	0.006
9	1%	2.7640	31.108	-432.76	28269.9	-0.868	-10.38	4082.1	0.0006	0.999	0.016
	5%	2.2744	16.469	-154.84	11461.3	-0.756	-6.82	2314.2	0.0003	0.999	0.008
	10%	2.0373	11.309	-101.55	7072.7	-0.696	-6.06	1736.0	0.0002	0.999	0.006
10	1%	2.6539	30.619	-409.19	30074.1	-0.825	-18.54	4772.3	0.0006	0.999	0.014
	5%	2.1939	16.723	-171.91	12823.1	-0.728	-10.95	2717.5	0.0003	0.999	0.007
	10%	1.9706	11.816	-135.38	8400.4	-0.672	-9.32	2041.0	0.0002	0.999	0.006

Note: The response surface regression model is equation (8). The dependent variable is the simulated α-quantile of the test statistic. Separate regressions are run for each number k of weakly exogenous regressors $\mathbf{x}_t$ in equation (6). SE($\theta_{0,0}$) denotes the heteroskedasticity-robust standard error of the intercept, $\bar{R}^2$ the adjusted coefficient of determination, and RMSE the root mean square error.

Table 9: Response surface estimates, F-statistic, upper bound, case (iv)

k	α	$\theta_{0,0}$	$\theta_{1,0}$	$\theta_{2,0}$	$\theta_{3,0}$	$\theta_{1,1}$	$\theta_{2,1}$	$\theta_{3,1}$	SE($\theta_{0,0}$)	$\bar{R}^2$	RMSE
0	1%	8.2726	45.413	154.08	1124.1	-2.656	4.29	2511.0	0.0017	0.991	0.136
	5%	6.2605	21.046	65.64	69.9	-2.254	31.51	597.1	0.0008	0.985	0.070
	10%	5.3366	13.098	36.97	-82.9	-1.847	25.94	258.5	0.0006	0.975	0.052
1	1%	6.6057	49.213	-286.87	9705.8	-0.717	36.21	2279.4	0.0014	0.997	0.072
	5%	5.1415	23.731	-115.45	3583.0	-0.609	29.92	690.4	0.0006	0.996	0.035
	10%	4.4554	15.397	-84.94	2174.8	-0.506	20.79	378.8	0.0004	0.994	0.026
2	1%	5.7472	41.494	13.91	7775.4	-0.121	23.60	2880.7	0.0012	0.997	0.064
	5%	4.5616	20.327	57.22	2120.1	-0.185	20.73	1028.2	0.0005	0.997	0.027
	10%	3.9993	13.130	46.79	903.7	-0.173	14.56	599.0	0.0004	0.997	0.018
3	1%	5.2013	42.313	-108.26	12225.4	-0.141	35.73	2780.6	0.0009	0.998	0.047
	5%	4.1945	20.458	37.75	3664.3	-0.167	30.29	900.6	0.0004	0.998	0.021
	10%	3.7116	13.335	35.17	1883.5	-0.153	22.65	478.8	0.0003	0.998	0.014
4	1%	4.8255	39.532	7.21	13681.4	-0.109	60.74	1974.7	0.0008	0.999	0.036
	5%	3.9383	20.035	73.80	4469.8	-0.115	39.43	596.0	0.0004	0.999	0.016
	10%	3.5111	13.189	59.54	2384.0	-0.106	28.44	282.2	0.0003	0.999	0.011
5	1%	4.5396	42.973	-233.57	20394.7	-0.079	46.84	2749.5	0.0009	0.999	0.035
	5%	3.7464	21.604	-2.07	6891.7	-0.079	32.41	955.1	0.0004	0.999	0.015
	10%	3.3616	14.197	22.15	3730.1	-0.074	24.21	497.3	0.0003	0.999	0.010
6	1%	4.3068	49.285	-669.32	30910.4	0.007	32.96	3384.3	0.0011	0.999	0.035
	5%	3.5944	23.756	-123.77	10370.3	-0.033	27.73	1188.4	0.0004	0.999	0.014
	10%	3.2438	15.485	-38.89	5652.1	-0.042	22.04	618.0	0.0003	0.999	0.009
7	1%	4.1366	49.091	-683.47	35272.2	0.009	27.49	3816.6	0.0009	0.999	0.029
	5%	3.4769	24.468	-154.95	12634.5	-0.028	24.97	1401.5	0.0004	0.999	0.012
	10%	3.1520	16.038	-53.45	6967.6	-0.034	19.97	758.1	0.0002	0.999	0.008
8	1%	3.9994	46.572	-572.10	38234.9	-0.029	47.98	3430.1	0.0009	0.999	0.027
	5%	3.3825	23.668	-104.39	13951.7	-0.041	34.42	1246.2	0.0004	0.999	0.011
	10%	3.0778	15.633	-18.33	7709.4	-0.039	25.81	678.8	0.0002	0.999	0.008
9	1%	3.8817	44.795	-359.86	37839.1	0.003	51.35	2725.5	0.0008	0.999	0.022
	5%	3.3007	23.597	-41.90	14429.5	-0.018	34.84	969.8	0.0004	0.999	0.009
	10%	3.0132	16.091	-6.92	8541.1	-0.020	25.40	526.2	0.0002	0.999	0.007
10	1%	3.7827	43.554	-234.69	39555.6	0.022	43.32	3603.2	0.0008	0.999	0.019
	5%	3.2321	23.516	2.38	15498.7	-0.004	30.06	1443.9	0.0004	0.999	0.009
	10%	2.9597	16.097	26.02	9120.7	-0.010	22.36	832.8	0.0002	0.999	0.006

Note: The response surface regression model is equation (8). The dependent variable is the simulated α-quantile of the test statistic. Separate regressions are run for each number k of weakly exogenous regressors $\mathbf{x}_t$ in equation (6). SE($\theta_{0,0}$) denotes the heteroskedasticity-robust standard error of the intercept, $\bar{R}^2$ the adjusted coefficient of determination, and RMSE the root mean square error.

Table 10: Response surface estimates, F-statistic, lower bound, case (v)

k	α	$\theta_{0,0}$	$\theta_{1,0}$	$\theta_{2,0}$	$\theta_{3,0}$	$\theta_{1,1}$	$\theta_{2,1}$	$\theta_{3,1}$	SE($\theta_{0,0}$)	$\bar{R}^2$	RMSE
0	1%	15.6672	74.372	185.49	1121.9	-8.703	87.05	3197.9	0.0041	0.976	0.327
	5%	11.6378	31.535	31.60	229.5	-7.003	98.25	346.7	0.0021	0.942	0.174
	10%	9.7837	17.880	-1.41	123.6	-5.744	68.01	-4.3	0.0015	0.894	0.125
1	1%	8.6578	53.977	-386.80	11734.0	-3.184	72.38	1673.8	0.0021	0.992	0.115
	5%	6.5535	23.744	-188.44	4752.8	-2.584	51.94	271.9	0.0010	0.985	0.061
	10%	5.5742	14.234	-145.42	3046.8	-2.227	35.98	73.9	0.0007	0.972	0.046
2	1%	6.3327	35.190	126.81	6172.7	-1.852	30.34	1984.7	0.0015	0.994	0.077
	5%	4.8627	14.956	118.92	1272.0	-1.599	25.70	524.5	0.0007	0.991	0.039
	10%	4.1747	8.269	97.39	290.0	-1.435	20.04	208.0	0.0005	0.986	0.028
3	1%	5.1477	33.218	-7.67	9634.1	-1.585	25.23	1833.6	0.0010	0.997	0.048
	5%	4.0046	14.191	82.85	2586.3	-1.356	24.63	412.6	0.0004	0.996	0.023
	10%	3.4649	8.117	75.53	1137.6	-1.215	20.17	111.4	0.0003	0.994	0.017
4	1%	4.4331	28.756	100.06	9858.5	-1.374	38.23	1147.5	0.0008	0.998	0.034
	5%	3.4833	13.035	118.45	2787.8	-1.151	24.95	241.6	0.0004	0.998	0.016
	10%	3.0333	7.755	98.15	1222.3	-1.029	18.30	40.4	0.0003	0.997	0.011
5	1%	3.9439	31.261	-148.35	15056.8	-1.226	14.56	2086.4	0.0008	0.999	0.029
	5%	3.1298	14.459	22.61	4762.9	-1.022	11.20	751.8	0.0003	0.999	0.012
	10%	2.7420	8.810	40.12	2417.6	-0.922	9.44	379.2	0.0002	0.999	0.009
6	1%	3.5836	34.980	-478.71	22258.5	-1.072	-1.42	2768.6	0.0009	0.999	0.027
	5%	2.8731	15.844	-86.21	7309.6	-0.921	4.56	1078.1	0.0003	0.999	0.011
	10%	2.5309	9.862	-25.60	3913.4	-0.836	4.13	638.2	0.0002	0.999	0.007
7	1%	3.3228	34.048	-499.60	24964.6	-1.016	-8.14	3293.5	0.0007	0.999	0.022
	5%	2.6822	16.125	-121.73	8835.6	-0.868	-0.70	1450.6	0.0003	0.999	0.009
	10%	2.3730	10.338	-57.36	4993.3	-0.790	0.58	903.9	0.0002	0.999	0.006
8	1%	3.1167	32.784	-499.57	27638.0	-0.968	-2.71	3415.5	0.0007	0.999	0.019
	5%	2.5318	16.062	-140.03	10303.2	-0.823	0.13	1630.4	0.0003	0.999	0.008
	10%	2.2488	10.504	-74.45	5983.0	-0.749	0.19	1082.3	0.0002	0.999	0.006
9	1%	2.9520	30.647	-381.70	27841.5	-0.918	-1.10	3521.8	0.0006	0.999	0.016
	5%	2.4111	15.672	-118.12	10965.9	-0.785	-0.25	1845.2	0.0003	0.999	0.007
	10%	2.1491	10.421	-67.38	6526.9	-0.716	-0.88	1330.0	0.0002	0.999	0.005
10	1%	2.8148	29.849	-334.35	29037.9	-0.867	-9.39	4222.0	0.0006	0.999	0.014
	5%	2.3108	15.822	-126.64	12126.2	-0.749	-4.91	2283.5	0.0003	0.999	0.007
	10%	2.0663	10.804	-93.09	7703.5	-0.686	-3.89	1639.1	0.0002	0.999	0.005

Note: The response surface regression model is equation (8). The dependent variable is the simulated α-quantile of the test statistic. Separate regressions are run for each number k of weakly exogenous regressors $\mathbf{x}_t$ in equation (6). SE($\theta_{0,0}$) denotes the heteroskedasticity-robust standard error of the intercept, $\bar{R}^2$ the adjusted coefficient of determination, and RMSE the root mean square error.

Table 11: Response surface estimates, F-statistic, upper bound, case (v)

k	α	$\theta_{0,0}$	$\theta_{1,0}$	$\theta_{2,0}$	$\theta_{3,0}$	$\theta_{1,1}$	$\theta_{2,1}$	$\theta_{3,1}$	SE($\theta_{0,0}$)	$\bar{R}^2$	RMSE
0	1%	15.6672	74.372	185.49	1121.9	-8.703	87.05	3197.9	0.0041	0.976	0.327
	5%	11.6378	31.535	31.60	229.5	-7.003	98.25	346.7	0.0021	0.942	0.174
	10%	9.7837	17.880	-1.41	123.6	-5.744	68.01	-4.3	0.0015	0.894	0.125
1	1%	9.4757	66.489	-508.98	13578.6	-1.643	54.31	2765.4	0.0021	0.995	0.116
	5%	7.2736	31.008	-258.62	5500.5	-1.410	38.54	804.6	0.0010	0.991	0.062
	10%	6.2405	19.323	-195.54	3484.2	-1.229	22.90	471.9	0.0007	0.982	0.049
2	1%	7.3794	49.423	14.60	8348.6	-0.346	20.79	3530.4	0.0016	0.997	0.080
	5%	5.7921	23.183	50.86	2084.2	-0.427	16.13	1328.6	0.0007	0.996	0.036
	10%	5.0387	14.418	30.44	884.8	-0.417	8.50	828.5	0.0005	0.994	0.026
3	1%	6.2934	47.128	-98.49	12772.6	-0.322	43.34	3073.3	0.0011	0.998	0.055
	5%	5.0280	22.005	30.71	3780.0	-0.343	33.73	991.0	0.0005	0.998	0.024
	10%	4.4209	13.666	28.22	1876.3	-0.319	23.69	545.6	0.0004	0.997	0.017
4	1%	5.6231	43.203	-2.49	14546.8	-0.220	67.63	2136.5	0.0010	0.998	0.041
	5%	4.5535	21.223	53.59	4805.8	-0.222	41.27	675.8	0.0004	0.999	0.018
	10%	4.0376	13.548	34.80	2644.2	-0.210	28.11	360.6	0.0003	0.998	0.012
5	1%	5.1587	45.567	-235.69	21158.0	-0.168	54.40	2814.2	0.0010	0.999	0.039
	5%	4.2277	22.329	-19.03	7209.9	-0.156	34.58	994.8	0.0004	0.999	0.016
	10%	3.7757	14.209	3.89	3939.3	-0.149	24.74	534.9	0.0003	0.999	0.011
6	1%	4.8057	51.615	-691.45	31924.2	-0.058	39.12	3419.8	0.0011	0.999	0.038
	5%	3.9860	24.135	-135.46	10634.7	-0.091	30.11	1191.8	0.0004	0.999	0.015
	10%	3.5826	15.234	-50.44	5789.7	-0.100	22.94	629.4	0.0003	0.999	0.010
7	1%	4.5507	51.076	-714.13	36444.1	-0.034	30.83	3916.8	0.0010	0.999	0.031
	5%	3.8043	24.729	-167.90	12906.1	-0.068	25.95	1427.3	0.0004	0.999	0.012
	10%	3.4363	15.807	-70.96	7194.4	-0.074	19.74	792.0	0.0003	0.999	0.008
8	1%	4.3518	48.107	-597.71	39355.8	-0.058	49.73	3573.6	0.0009	0.999	0.028
	5%	3.6625	23.853	-122.07	14288.0	-0.071	34.41	1301.0	0.0004	0.999	0.012
	10%	3.3218	15.342	-35.71	7944.8	-0.070	25.04	722.4	0.0003	0.999	0.008
9	1%	4.1866	46.029	-378.60	38771.0	-0.021	53.53	2803.7	0.0009	0.999	0.023
	5%	3.5444	23.693	-62.05	14837.0	-0.043	34.68	1031.3	0.0004	0.999	0.010
	10%	3.2264	15.744	-27.02	8844.7	-0.045	24.54	586.4	0.0003	0.999	0.007
10	1%	4.0495	45.024	-287.62	41126.0	0.005	43.67	3743.9	0.0008	0.999	0.020
	5%	3.4469	23.628	-27.22	16086.0	-0.025	29.75	1506.3	0.0004	0.999	0.009
	10%	3.1484	15.668	8.66	9388.1	-0.031	21.73	873.8	0.0003	0.999	0.006

Note: The response surface regression model is equation (8). The dependent variable is the simulated α-quantile of the test statistic. Separate regressions are run for each number k of weakly exogenous regressors $\mathbf{x}_t$ in equation (6). SE($\theta_{0,0}$) denotes the heteroskedasticity-robust standard error of the intercept, $\bar{R}^2$ the adjusted coefficient of determination, and RMSE the root mean square error.

Table 12: Response surface estimates, t-statistic, lower bound, case (i)

k	α	$\theta_{0,0}$	$\theta_{1,0}$	$\theta_{2,0}$	$\theta_{3,0}$	$\theta_{1,1}$	$\theta_{2,1}$	$\theta_{3,1}$	SE($\theta_{0,0}$)	$\bar{R}^2$	RMSE
0	1%	-2.5570	-4.228	17.84	-177.5	0.536	-12.54	1.3	0.0003	0.977	0.015
	5%	-1.9356	-1.794	27.57	-257.0	0.477	-11.72	63.6	0.0002	0.919	0.008
	10%	-1.6133	-0.889	28.87	-263.2	0.427	-10.13	63.4	0.0001	0.812	0.006
1	1%	-2.5601	-3.931	13.74	-347.2	0.267	-14.04	12.1	0.0003	0.983	0.014
	5%	-1.9372	-1.621	25.65	-335.6	0.243	-9.16	52.5	0.0001	0.947	0.007
	10%	-1.6145	-0.751	29.32	-332.2	0.237	-6.68	47.1	0.0001	0.758	0.006
2	1%	-2.5597	-4.083	26.05	-691.1	0.162	-12.93	-11.4	0.0003	0.984	0.014
	5%	-1.9373	-1.682	32.07	-480.2	0.163	-7.05	28.6	0.0002	0.948	0.007
	10%	-1.6145	-0.773	34.74	-425.3	0.170	-4.09	17.9	0.0001	0.773	0.006
3	1%	-2.5618	-3.574	2.53	-629.1	0.136	-14.66	35.5	0.0003	0.981	0.013
	5%	-1.9386	-1.385	19.98	-401.7	0.137	-6.99	40.5	0.0002	0.939	0.007
	10%	-1.6157	-0.498	26.13	-351.6	0.150	-3.60	20.5	0.0001	0.781	0.005
4	1%	-2.5614	-3.616	0.82	-770.8	0.092	-11.73	-39.5	0.0003	0.981	0.013
	5%	-1.9387	-1.409	22.33	-486.7	0.115	-5.63	15.0	0.0002	0.938	0.007
	10%	-1.6160	-0.499	32.12	-464.8	0.139	-2.94	10.3	0.0001	0.809	0.005
5	1%	-2.5637	-3.065	-31.75	-536.9	0.094	-13.35	8.0	0.0004	0.979	0.012
	5%	-1.9393	-1.264	14.86	-443.1	0.106	-5.17	18.0	0.0002	0.927	0.007
	10%	-1.6162	-0.472	34.07	-507.5	0.128	-1.80	-3.5	0.0002	0.797	0.005
6	1%	-2.5630	-3.175	-30.45	-776.8	0.072	-11.72	-30.6	0.0004	0.980	0.012
	5%	-1.9396	-1.196	12.88	-486.8	0.097	-4.65	14.3	0.0002	0.933	0.007
	10%	-1.6168	-0.354	33.32	-533.1	0.126	-1.74	5.1	0.0002	0.828	0.005
7	1%	-2.5635	-3.220	-27.84	-1090.6	0.074	-12.67	-12.1	0.0004	0.982	0.012
	5%	-1.9402	-1.073	11.31	-571.5	0.098	-5.69	50.1	0.0002	0.938	0.007
	10%	-1.6173	-0.202	31.79	-557.9	0.124	-2.02	25.2	0.0002	0.832	0.005
8	1%	-2.5631	-3.528	-7.83	-1696.0	0.067	-11.43	-68.4	0.0004	0.984	0.013
	5%	-1.9402	-1.173	20.72	-803.3	0.091	-4.42	15.7	0.0002	0.946	0.007
	10%	-1.6173	-0.196	37.97	-689.4	0.118	-0.96	3.5	0.0002	0.845	0.005
9	1%	-2.5637	-3.277	-22.08	-1847.4	0.064	-12.25	-50.4	0.0004	0.981	0.012
	5%	-1.9407	-1.082	19.38	-892.4	0.091	-4.62	26.0	0.0002	0.935	0.007
	10%	-1.6177	-0.179	45.41	-855.5	0.120	-0.99	3.4	0.0002	0.854	0.005
10	1%	-2.5639	-3.316	-18.55	-2274.8	0.062	-12.64	7.4	0.0005	0.977	0.012
	5%	-1.9410	-0.999	18.75	-1000.6	0.090	-5.32	81.1	0.0002	0.918	0.007
	10%	-1.6179	-0.151	52.67	-1031.8	0.120	-1.32	38.4	0.0002	0.838	0.005

Note: The response surface regression model is equation (8). The dependent variable is the simulated α-quantile of the test statistic. Separate regressions are run for each number k of weakly exogenous regressors $\mathbf{x}_t$ in equation (6). SE($\theta_{0,0}$) denotes the heteroskedasticity-robust standard error of the intercept, $\bar{R}^2$ the adjusted coefficient of determination, and RMSE the root mean square error.

Table 13: Response surface estimates, t-statistic, upper bound, case (i)

k	α	$\theta_{0,0}$	$\theta_{1,0}$	$\theta_{2,0}$	$\theta_{3,0}$	$\theta_{1,1}$	$\theta_{2,1}$	$\theta_{3,1}$	SE($\theta_{0,0}$)	$\bar{R}^2$	RMSE
0	1%	-2.5570	-4.228	17.84	-177.5	0.536	-12.54	1.3	0.0003	0.977	0.015
	5%	-1.9356	-1.794	27.57	-257.0	0.477	-11.72	63.6	0.0002	0.919	0.008
	10%	-1.6133	-0.889	28.87	-263.2	0.427	-10.13	63.4	0.0001	0.812	0.006
1	1%	-3.2084	-6.088	13.10	-388.0	0.341	-13.90	-53.9	0.0003	0.989	0.016
	5%	-2.5919	-2.736	22.11	-338.2	0.336	-9.88	44.5	0.0002	0.971	0.009
	10%	-2.2631	-1.555	23.72	-314.9	0.333	-7.78	61.0	0.0001	0.905	0.008
2	1%	-3.6158	-8.125	50.03	-994.4	0.263	-6.47	-294.8	0.0003	0.992	0.016
	5%	-3.0024	-3.498	39.12	-541.5	0.317	-2.75	-127.3	0.0002	0.974	0.010
	10%	-2.6728	-1.854	35.56	-401.7	0.352	-0.78	-92.8	0.0002	0.924	0.010
3	1%	-3.9436	-7.563	-15.28	-449.8	0.361	-11.84	-294.7	0.0003	0.993	0.015
	5%	-3.3268	-2.848	6.46	-130.7	0.416	-3.31	-206.9	0.0002	0.975	0.010
	10%	-2.9950	-1.143	16.79	-73.3	0.461	0.17	-202.6	0.0002	0.946	0.010
4	1%	-4.2179	-8.454	14.11	-1172.9	0.455	-14.42	-383.7	0.0004	0.994	0.015
	5%	-3.6006	-2.813	28.79	-528.0	0.550	-7.95	-209.3	0.0002	0.976	0.011
	10%	-3.2672	-0.772	39.91	-411.0	0.613	-5.05	-183.6	0.0002	0.956	0.012
5	1%	-4.4577	-9.614	60.84	-2104.3	0.507	-10.49	-613.0	0.0004	0.992	0.016
	5%	-3.8367	-3.803	109.24	-1648.5	0.599	-1.01	-499.2	0.0003	0.962	0.013
	10%	-3.5015	-1.583	130.40	-1590.4	0.665	2.85	-498.5	0.0004	0.947	0.014
6	1%	-4.6757	-10.385	116.71	-3354.8	0.587	-10.28	-829.7	0.0005	0.992	0.017
	5%	-4.0551	-3.525	150.65	-2381.0	0.721	-4.64	-600.1	0.0004	0.965	0.014
	10%	-3.7195	-0.854	167.29	-2184.4	0.806	-2.55	-554.3	0.0004	0.959	0.015
7	1%	-4.8779	-10.502	156.73	-4534.7	0.679	-13.75	-980.6	0.0005	0.992	0.017
	5%	-4.2556	-2.905	183.40	-3046.5	0.815	-7.00	-733.8	0.0004	0.962	0.016
	10%	-3.9191	0.112	198.56	-2717.8	0.904	-4.72	-686.2	0.0004	0.963	0.016
8	1%	-5.0635	-11.403	249.18	-6523.7	0.731	-7.50	-1453.6	0.0006	0.992	0.019
	5%	-4.4408	-2.487	231.22	-3900.0	0.875	-1.77	-1128.6	0.0005	0.960	0.017
	10%	-4.1036	0.922	240.35	-3336.5	0.972	0.05	-1065.4	0.0005	0.965	0.019
9	1%	-5.2413	-11.169	303.31	-8191.6	0.831	-12.18	-1646.7	0.0006	0.990	0.020
	5%	-4.6169	-1.830	292.46	-5127.8	0.992	-7.77	-1198.8	0.0006	0.961	0.019
	10%	-4.2789	1.662	315.56	-4623.3	1.098	-6.40	-1112.7	0.0007	0.968	0.021
10	1%	-5.4088	-10.274	331.14	-9654.3	0.883	-7.08	-2554.5	0.0007	0.986	0.020
	5%	-4.7837	-0.179	321.40	-6140.7	1.058	-4.90	-2101.8	0.0007	0.962	0.020
	10%	-4.4453	3.781	337.64	-5412.7	1.173	-5.17	-2005.1	0.0008	0.971	0.021

Note: The response surface regression model is equation (8). The dependent variable is the simulated α-quantile of the test statistic. Separate regressions are run for each number k of weakly exogenous regressors $\mathbf{x}_t$ in equation (6). SE($\theta_{0,0}$) denotes the heteroskedasticity-robust standard error of the intercept, $\bar{R}^2$ the adjusted coefficient of determination, and RMSE the root mean square error.

Table 14: Response surface estimates, t-statistic, lower bound, case (iii)

k	α	$\theta_{0,0}$	$\theta_{1,0}$	$\theta_{2,0}$	$\theta_{3,0}$	$\theta_{1,1}$	$\theta_{2,1}$	$\theta_{3,1}$	SE($\theta_{0,0}$)	$\bar{R}^2$	RMSE
0	1%	-3.4298	-6.418	-32.65	332.8	0.683	-3.74	-270.9	0.0003	0.982	0.023
	5%	-2.8619	-2.902	-10.94	158.3	0.671	-6.05	-77.9	0.0002	0.948	0.015
	10%	-2.5672	-1.666	-3.56	77.2	0.625	-5.09	-31.3	0.0001	0.900	0.012
1	1%	-3.4290	-6.987	24.00	-853.0	0.539	-17.17	-52.3	0.0003	0.989	0.018
	5%	-2.8609	-3.076	24.74	-531.0	0.540	-10.99	36.4	0.0002	0.965	0.012
	10%	-2.5663	-1.667	25.81	-441.9	0.544	-7.87	45.4	0.0002	0.932	0.010
2	1%	-3.4266	-6.721	1.31	-559.3	0.343	-2.29	-388.9	0.0004	0.989	0.017
	5%	-2.8595	-2.620	3.01	-158.8	0.411	0.27	-191.6	0.0002	0.972	0.010
	10%	-2.5653	-1.147	8.09	-104.4	0.454	1.56	-138.8	0.0002	0.960	0.008
3	1%	-3.4302	-5.918	-8.97	-921.4	0.410	-13.32	-146.2	0.0003	0.991	0.014
	5%	-2.8623	-1.795	-9.54	-204.4	0.461	-6.22	-42.5	0.0002	0.979	0.008
	10%	-2.5679	-0.265	-5.45	-53.7	0.502	-3.39	-21.3	0.0001	0.976	0.007
4	1%	-3.4308	-5.528	-11.93	-1222.7	0.402	-13.59	-152.3	0.0003	0.990	0.014
	5%	-2.8630	-1.488	-0.16	-475.9	0.472	-7.04	-20.5	0.0002	0.982	0.008
	10%	-2.5687	0.087	5.42	-277.9	0.521	-4.10	1.0	0.0002	0.985	0.006
5	1%	-3.4302	-5.498	-2.50	-1689.0	0.388	-11.04	-195.9	0.0004	0.992	0.012
	5%	-2.8625	-1.280	8.71	-711.6	0.463	-5.29	-34.5	0.0002	0.984	0.007
	10%	-2.5683	0.442	12.29	-427.6	0.514	-2.84	2.6	0.0002	0.986	0.006
6	1%	-3.4304	-5.296	1.52	-2132.2	0.387	-10.69	-190.7	0.0004	0.992	0.012
	5%	-2.8626	-1.066	17.75	-948.2	0.461	-3.33	-66.5	0.0002	0.987	0.007
	10%	-2.5684	0.673	24.53	-619.3	0.514	-0.28	-44.6	0.0002	0.989	0.006
7	1%	-3.4299	-5.292	27.71	-3022.0	0.387	-10.41	-239.7	0.0004	0.993	0.012
	5%	-2.8626	-0.622	20.17	-1159.7	0.457	-2.94	-74.4	0.0002	0.986	0.007
	10%	-2.5685	1.295	21.42	-665.1	0.511	-0.26	-34.8	0.0002	0.989	0.005
8	1%	-3.4301	-5.097	37.53	-3660.4	0.382	-8.30	-293.3	0.0004	0.991	0.012
	5%	-2.8627	-0.419	32.93	-1475.8	0.456	0.07	-168.2	0.0002	0.986	0.007
	10%	-2.5686	1.533	36.16	-905.1	0.510	3.29	-140.4	0.0002	0.990	0.006
9	1%	-3.4300	-5.032	53.67	-4395.3	0.381	-6.21	-369.7	0.0004	0.990	0.012
	5%	-2.8633	-0.064	42.72	-1811.6	0.463	1.77	-284.3	0.0002	0.986	0.007
	10%	-2.5694	1.994	43.55	-1081.3	0.520	4.52	-230.9	0.0002	0.990	0.006
10	1%	-3.4314	-4.262	22.34	-4335.8	0.388	-7.73	-299.3	0.0005	0.987	0.012
	5%	-2.8644	0.640	28.19	-1730.2	0.475	-1.75	-81.1	0.0003	0.984	0.007
	10%	-2.5701	2.596	41.47	-1115.0	0.533	1.45	-55.3	0.0002	0.990	0.006

Note: The response surface regression model is equation (8). The dependent variable is the simulated α-quantile of the test statistic. Separate regressions are run for each number k of weakly exogenous regressors $\mathbf{x}_t$ in equation (6). SE($\theta_{0,0}$) denotes the heteroskedasticity-robust standard error of the intercept, $\bar{R}^2$ the adjusted coefficient of determination, and RMSE the root mean square error.

Table 15: Response surface estimates, t-statistic, upper bound, case (iii)

k	α	$\theta_{0,0}$	$\theta_{1,0}$	$\theta_{2,0}$	$\theta_{3,0}$	$\theta_{1,1}$	$\theta_{2,1}$	$\theta_{3,1}$	SE($\theta_{0,0}$)	$\bar{R}^2$	RMSE
0	1%	-3.4298	-6.418	-32.65	332.8	0.683	-3.74	-270.9	0.0003	0.982	0.023
	5%	-2.8619	-2.902	-10.94	158.3	0.671	-6.05	-77.9	0.0002	0.948	0.015
	10%	-2.5672	-1.666	-3.56	77.2	0.625	-5.09	-31.3	0.0001	0.900	0.012
1	1%	-3.7946	-8.954	39.61	-1093.4	0.527	-19.61	-62.8	0.0004	0.990	0.020
	5%	-3.2140	-4.244	36.61	-684.5	0.517	-12.19	36.5	0.0002	0.968	0.014
	10%	-2.9080	-2.556	38.55	-594.4	0.523	-8.70	47.6	0.0002	0.915	0.013
2	1%	-4.0902	-10.288	25.15	-835.1	0.336	0.21	-588.7	0.0004	0.993	0.019
	5%	-3.5031	-4.818	30.89	-401.7	0.420	3.00	-332.0	0.0002	0.976	0.014
	10%	-3.1906	-2.804	36.87	-327.2	0.474	4.65	-266.7	0.0002	0.936	0.013
3	1%	-4.3540	-10.586	19.00	-1163.3	0.471	-9.57	-522.6	0.0004	0.995	0.017
	5%	-3.7596	-4.441	19.67	-283.1	0.541	-1.04	-357.6	0.0003	0.979	0.013
	10%	-3.4423	-2.066	23.42	-65.9	0.598	2.57	-333.3	0.0003	0.950	0.013
4	1%	-4.5868	-11.348	56.77	-2131.2	0.565	-13.23	-622.0	0.0004	0.995	0.016
	5%	-3.9877	-4.855	77.50	-1176.9	0.674	-5.09	-421.7	0.0003	0.979	0.013
	10%	-3.6664	-2.453	97.35	-1068.5	0.748	-1.39	-397.2	0.0003	0.961	0.014
5	1%	-4.7974	-12.275	112.58	-3378.9	0.633	-13.40	-803.0	0.0004	0.995	0.017
	5%	-4.1950	-4.786	125.13	-2009.3	0.766	-8.44	-493.3	0.0004	0.974	0.014
	10%	-3.8713	-1.912	140.27	-1752.3	0.856	-6.58	-420.6	0.0004	0.954	0.015
6	1%	-4.9901	-13.760	212.04	-5299.6	0.681	-5.99	-1243.5	0.0005	0.994	0.018
	5%	-4.3843	-5.767	230.97	-3542.9	0.816	2.52	-1013.7	0.0005	0.970	0.017
	10%	-4.0581	-2.706	253.14	-3243.9	0.909	5.70	-978.1	0.0005	0.960	0.018
7	1%	-5.1719	-14.050	276.98	-7038.5	0.747	-6.57	-1545.9	0.0005	0.994	0.018
	5%	-4.5637	-4.831	256.85	-4194.2	0.880	1.42	-1232.6	0.0005	0.963	0.017
	10%	-4.2357	-1.279	267.61	-3585.7	0.973	4.34	-1176.7	0.0005	0.957	0.018
8	1%	-5.3435	-14.516	356.92	-9008.6	0.798	2.77	-2236.6	0.0006	0.991	0.020
	5%	-4.7320	-5.272	374.13	-6160.3	0.944	13.00	-2025.8	0.0006	0.963	0.019
	10%	-4.4025	-1.489	389.40	-5457.4	1.046	15.96	-1990.3	0.0006	0.965	0.021
9	1%	-5.5100	-13.404	385.52	-10621.9	0.889	3.11	-3208.2	0.0007	0.990	0.020
	5%	-4.8970	-3.385	399.12	-7200.6	1.054	12.52	-3104.8	0.0007	0.969	0.019
	10%	-4.5665	0.748	417.04	-6421.4	1.166	15.13	-3144.6	0.0007	0.973	0.021
10	1%	-5.6635	-14.155	537.48	-14372.1	0.970	-2.52	-3136.1	0.0009	0.985	0.022
	5%	-5.0495	-3.069	521.85	-9850.3	1.154	1.38	-2695.3	0.0008	0.962	0.022
	10%	-4.7175	1.126	560.01	-9287.0	1.275	1.94	-2595.8	0.0009	0.970	0.023

Note: The response surface regression model is equation (8). The dependent variable is the simulated α-quantile of the test statistic. Separate regressions are run for each number k of weakly exogenous regressors $\mathbf{x}_t$ in equation (6). SE($\theta_{0,0}$) denotes the heteroskedasticity-robust standard error of the intercept, $\bar{R}^2$ the adjusted coefficient of determination, and RMSE the root mean square error.

Table 16: Response surface estimates, t-statistic, lower bound, case (v)

k	α	$\theta_{0,0}$	$\theta_{1,0}$	$\theta_{2,0}$	$\theta_{3,0}$	$\theta_{1,1}$	$\theta_{2,1}$	$\theta_{3,1}$	SE($\theta_{0,0}$)	$\bar{R}^2$	RMSE
0	1%	-3.9594	-9.262	-16.02	-72.9	1.187	-20.52	-220.4	0.0005	0.978	0.036
	5%	-3.4117	-4.616	-2.02	-19.4	1.050	-16.42	-22.2	0.0003	0.944	0.024
	10%	-3.1280	-2.862	0.71	-3.5	0.929	-11.23	1.2	0.0002	0.898	0.019
1	1%	-3.9525	-10.789	91.02	-1992.6	0.763	-19.87	-192.9	0.0004	0.987	0.025
	5%	-3.4064	-5.322	66.45	-1137.3	0.743	-11.24	-60.1	0.0003	0.965	0.018
	10%	-3.1237	-3.233	57.56	-856.0	0.733	-6.57	-44.6	0.0002	0.935	0.015
2	1%	-3.9567	-8.224	-33.20	-563.7	0.588	-9.72	-426.3	0.0004	0.990	0.021
	5%	-3.4091	-3.358	-17.09	-37.8	0.635	-2.92	-227.2	0.0002	0.974	0.014
	10%	-3.1259	-1.510	-7.47	62.7	0.668	0.63	-185.5	0.0002	0.963	0.012
3	1%	-3.9600	-7.446	-30.31	-1196.7	0.659	-20.37	-201.9	0.0004	0.992	0.017
	5%	-3.4121	-2.419	-18.03	-307.4	0.706	-11.26	-43.7	0.0002	0.982	0.011
	10%	-3.1288	-0.458	-9.78	-96.7	0.743	-7.16	-13.2	0.0002	0.977	0.009
4	1%	-3.9589	-7.344	-9.52	-1960.8	0.636	-20.39	-109.1	0.0004	0.993	0.015
	5%	-3.4112	-2.317	9.20	-834.6	0.699	-9.10	-38.7	0.0002	0.989	0.009
	10%	-3.1281	-0.368	23.37	-607.6	0.747	-4.28	-40.6	0.0002	0.990	0.008
5	1%	-3.9589	-7.155	11.82	-2778.2	0.638	-18.94	-156.5	0.0004	0.994	0.014
	5%	-3.4119	-1.604	8.85	-1031.5	0.704	-9.23	-8.6	0.0002	0.990	0.008
	10%	-3.1287	0.492	19.96	-661.3	0.754	-4.65	3.8	0.0002	0.991	0.007
6	1%	-3.9584	-7.034	38.21	-3730.7	0.630	-17.16	-186.0	0.0004	0.995	0.013
	5%	-3.4121	-1.180	24.21	-1435.6	0.711	-8.13	-18.0	0.0002	0.993	0.007
	10%	-3.1295	1.118	29.16	-861.8	0.769	-4.06	6.1	0.0002	0.993	0.006
7	1%	-3.9575	-6.939	63.56	-4671.5	0.607	-11.15	-349.3	0.0004	0.994	0.013
	5%	-3.4113	-0.744	35.01	-1754.3	0.691	-3.64	-109.3	0.0002	0.991	0.007
	10%	-3.1285	1.662	39.30	-1050.0	0.747	0.37	-76.8	0.0002	0.993	0.006
8	1%	-3.9581	-6.365	69.77	-5435.3	0.608	-9.92	-416.4	0.0004	0.993	0.012
	5%	-3.4114	-0.427	60.01	-2328.1	0.692	0.17	-246.5	0.0002	0.992	0.007
	10%	-3.1286	2.044	64.10	-1472.5	0.750	4.62	-223.6	0.0002	0.994	0.006
9	1%	-3.9587	-5.852	73.44	-6090.6	0.613	-9.47	-403.1	0.0005	0.992	0.012
	5%	-3.4122	0.126	75.17	-2792.7	0.703	1.26	-328.0	0.0003	0.992	0.007
	10%	-3.1295	2.642	83.23	-1877.5	0.764	6.09	-348.5	0.0002	0.994	0.007
10	1%	-3.9602	-4.887	48.23	-6285.7	0.623	-11.39	-298.2	0.0005	0.989	0.012
	5%	-3.4131	0.980	70.39	-2937.1	0.718	-1.93	-134.1	0.0003	0.991	0.007
	10%	-3.1305	3.549	82.88	-1981.9	0.783	2.01	-106.7	0.0003	0.994	0.007

Note: The response surface regression model is equation (8). The dependent variable is the simulated α-quantile of the test statistic. Separate regressions are run for each number k of weakly exogenous regressors $\mathbf{x}_t$ in equation (6). SE($\theta_{0,0}$) denotes the heteroskedasticity-robust standard error of the intercept, $\bar{R}^2$ the adjusted coefficient of determination, and RMSE the root mean square error.

Table 17: Response surface estimates, t-statistic, upper bound, case (v)

k	α	$\theta_{0,0}$	$\theta_{1,0}$	$\theta_{2,0}$	$\theta_{3,0}$	$\theta_{1,1}$	$\theta_{2,1}$	$\theta_{3,1}$	SE($\theta_{0,0}$)	$\bar{R}^2$	RMSE
0	1%	-3.9594	-9.262	-16.02	-72.9	1.187	-20.52	-220.4	0.0005	0.978	0.036
	5%	-3.4117	-4.616	-2.02	-19.4	1.050	-16.42	-22.2	0.0003	0.944	0.024
	10%	-3.1280	-2.862	0.71	-3.5	0.929	-11.23	1.2	0.0002	0.898	0.019
1	1%	-4.2423	-12.982	107.28	-2216.0	0.673	-17.64	-275.4	0.0004	0.989	0.027
	5%	-3.6829	-6.773	80.14	-1266.2	0.654	-8.56	-123.6	0.0003	0.970	0.019
	10%	-3.3893	-4.445	73.88	-999.1	0.655	-3.71	-107.6	0.0002	0.932	0.017
2	1%	-4.4946	-12.270	4.22	-983.3	0.496	-2.92	-719.4	0.0004	0.993	0.022
	5%	-3.9239	-6.180	23.99	-360.5	0.559	5.01	-486.2	0.0003	0.979	0.016
	10%	-3.6222	-3.862	37.59	-269.6	0.611	8.67	-435.5	0.0002	0.951	0.016
3	1%	-4.7214	-12.803	22.32	-1718.9	0.623	-12.39	-699.5	0.0004	0.995	0.019
	5%	-4.1427	-6.012	46.11	-792.1	0.702	-3.23	-464.3	0.0003	0.983	0.014
	10%	-3.8352	-3.368	59.42	-570.4	0.764	1.06	-421.5	0.0003	0.958	0.014
4	1%	-4.9232	-14.522	117.59	-3548.8	0.663	-9.17	-957.2	0.0004	0.995	0.017
	5%	-4.3380	-7.476	163.07	-2451.3	0.767	3.86	-847.6	0.0003	0.982	0.015
	10%	-4.0260	-4.703	188.60	-2259.9	0.845	9.50	-861.9	0.0004	0.969	0.015
5	1%	-5.1123	-15.288	181.92	-5110.2	0.735	-9.81	-1176.4	0.0005	0.995	0.018
	5%	-4.5230	-7.181	207.53	-3285.3	0.861	-0.09	-919.4	0.0004	0.979	0.016
	10%	-4.2074	-3.989	229.31	-2918.5	0.948	4.18	-886.2	0.0004	0.960	0.017
6	1%	-5.2898	-16.099	275.37	-7277.1	0.817	-10.54	-1474.8	0.0005	0.996	0.018
	5%	-4.6979	-6.718	262.66	-4341.6	0.969	-2.54	-1149.5	0.0005	0.979	0.017
	10%	-4.3801	-3.056	275.64	-3679.5	1.071	0.78	-1096.3	0.0005	0.967	0.018
7	1%	-5.4562	-16.433	347.30	-9217.5	0.862	-7.31	-1871.5	0.0006	0.994	0.019
	5%	-4.8601	-6.246	326.82	-5622.3	1.020	-1.69	-1432.3	0.0005	0.968	0.018
	10%	-4.5397	-2.227	339.39	-4803.6	1.125	0.88	-1354.9	0.0006	0.961	0.019
8	1%	-5.6130	-17.509	469.67	-11915.9	0.893	6.31	-2687.0	0.0007	0.990	0.021
	5%	-5.0123	-7.505	495.74	-8385.3	1.048	17.95	-2426.2	0.0007	0.964	0.022
	10%	-4.6894	-3.462	524.69	-7615.2	1.157	21.92	-2387.6	0.0007	0.965	0.023
9	1%	-5.7677	-16.761	551.99	-14837.3	0.981	7.39	-3806.7	0.0008	0.989	0.021
	5%	-5.1644	-6.340	607.68	-11312.1	1.152	20.63	-3819.9	0.0008	0.971	0.021
	10%	-4.8399	-2.040	648.28	-10633.6	1.269	24.58	-3881.2	0.0008	0.973	0.023
10	1%	-5.9121	-17.343	717.56	-19266.1	1.059	2.13	-3774.9	0.0010	0.984	0.023
	5%	-5.3060	-6.425	791.19	-15498.8	1.250	9.72	-3433.1	0.0010	0.964	0.024
	10%	-4.9800	-1.800	834.06	-14708.0	1.378	10.12	-3284.1	0.0011	0.969	0.025

Note: The response surface regression model is equation (8). The dependent variable is the simulated α-quantile of the test statistic. Separate regressions are run for each number k of weakly exogenous regressors $\mathbf{x}_t$ in equation (6). SE($\theta_{0,0}$) denotes the heteroskedasticity-robust standard error of the intercept, $\bar{R}^2$ the adjusted coefficient of determination, and RMSE the root mean square error.

Table 18: Response surface estimates, F-statistic, case (i)

	$\alpha = 1\%$		$\alpha = 5\%$		$\alpha = 10\%$	
	$I(0)$	$I(1)$	$I(0)$	$I(1)$	$I(0)$	$I(1)$
$\theta_{0,0,0}$	1.3696	2.4281	1.2942	2.3622	1.2370	2.2960
$\theta_{1,0,0}$	10.6537	14.0067	6.2409	7.9543	4.3366	5.3600
$\theta_{2,0,0}$	-13.6561	-26.0058	-8.8317	-15.9274	-6.7007	-11.8646
$\theta_{3,0,0}$	15.4161	31.9173	9.4594	18.1120	7.0777	13.0640
$\theta_{4,0,0}$	-6.7631	-15.2134	-3.9997	-8.2957	-2.9522	-5.8334
$\theta_{0,1,0}$	43.832	89.090	22.341	46.696	15.234	31.655
$\theta_{1,1,0}$	-300.208	-733.557	-141.425	-368.295	-91.727	-247.906
$\theta_{2,1,0}$	974.321	2450.577	426.737	1192.902	263.505	792.198
$\theta_{3,1,0}$	-1361.786	-3547.985	-577.890	-1701.901	-347.942	-1121.248
$\theta_{4,1,0}$	652.692	1734.590	272.626	826.596	161.970	542.515
$\theta_{0,2,0}$	452.19	878.73	186.33	360.06	98.79	205.25
$\theta_{0,3,0}$	-2057.4	-4987.2	-1060.6	-2279.6	-573.0	-1327.0
$\theta_{0,1,1}$	-0.753	-0.407	-0.572	-0.223	-0.495	-0.146
$\theta_{1,1,1}$	1.199	3.901	0.494	2.658	0.350	1.939
$\theta_{2,1,1}$	-9.034	-7.868	-6.079	-10.421	-4.562	-8.815
$\theta_{3,1,1}$	19.607	-3.609	14.778	10.738	11.604	11.147
$\theta_{4,1,1}$	-12.451	5.657	-9.339	-3.762	-7.324	-4.659
$\theta_{0,2,1}$	39.31	74.26	27.88	39.99	20.90	25.73
$\theta_{0,3,1}$	331.8	56.3	-76.3	-179.4	-107.6	-152.4
$\bar{R}^2$	0.9980	0.9934	0.9982	0.9927	0.9977	0.9898
RMSE	0.0769	0.1146	0.0344	0.0453	0.0230	0.0271

Note: The response surface regression model is equation (10). The dependent variable is the simulated α-quantile of the test statistic. $\bar{R}^2$ denotes the adjusted coefficient of determination, and RMSE the root mean square error.

Table 19: Response surface estimates, F-statistic, case (ii)

	$\alpha = 1\%$		$\alpha = 5\%$		$\alpha = 10\%$	
	$I(0)$	$I(1)$	$I(0)$	$I(1)$	$I(0)$	$I(1)$
$\theta_{0,0,0}$	1.4019	2.5281	1.3055	2.4053	1.2438	2.3247
$\theta_{1,0,0}$	12.2206	12.9344	8.0776	7.8819	6.2268	5.5885
$\theta_{2,0,0}$	-16.7218	-24.9204	-11.1424	-16.0068	-8.5678	-11.8062
$\theta_{3,0,0}$	16.2339	29.0801	10.7213	18.7103	8.2050	13.9175
$\theta_{4,0,0}$	-6.6301	-13.0084	-4.3245	-8.3088	-3.2954	-6.1863
$\theta_{0,1,0}$	39.756	79.524	21.222	44.059	14.650	30.617
$\theta_{1,1,0}$	-211.809	-555.426	-109.353	-302.872	-74.456	-210.728
$\theta_{2,1,0}$	606.821	1660.558	298.970	909.712	195.959	634.363
$\theta_{3,1,0}$	-772.210	-2238.762	-372.888	-1231.818	-239.173	-860.622
$\theta_{4,1,0}$	348.517	1048.841	166.626	578.749	105.581	404.890
$\theta_{0,2,0}$	555.39	997.78	249.93	439.08	147.87	263.95
$\theta_{0,3,0}$	-1810.3	-4949.6	-1216.5	-2627.9	-778.9	-1670.9
$\theta_{0,1,1}$	-0.612	-0.156	-0.532	-0.110	-0.482	-0.084
$\theta_{1,1,1}$	-2.983	-0.255	-2.052	0.670	-1.745	0.735
$\theta_{2,1,1}$	9.329	14.773	2.659	0.623	1.487	-2.007
$\theta_{3,1,1}$	-14.823	-45.998	-1.203	-10.996	0.963	-3.229
$\theta_{4,1,1}$	6.966	28.541	-0.427	7.841	-1.494	3.082
$\theta_{0,2,1}$	27.98	72.06	22.15	41.23	17.36	28.00
$\theta_{0,3,1}$	792.8	399.7	139.3	-32.8	20.5	-75.2
$\bar{R}^2$	0.9979	0.9931	0.9989	0.9951	0.9991	0.9954
RMSE	0.0728	0.1116	0.0315	0.0455	0.0206	0.0277

Note: The response surface regression model is equation (10). The dependent variable is the simulated α-quantile of the test statistic. $\bar{R}^2$ denotes the adjusted coefficient of determination, and RMSE the root mean square error.

Table 20: Response surface estimates, F-statistic, case (iii)

	α = 1%		α = 5%		α = 10%	
	$I(0)$	$I(1)$	$I(0)$	$I(1)$	$I(0)$	$I(1)$
$\theta_{0,0,0}$	1.3503	2.4703	1.2769	2.3748	1.2232	2.3014
$\theta_{1,0,0}$	13.3980	15.6809	8.8811	10.0441	6.8420	7.5100
$\theta_{2,0,0}$	-8.8477	-18.9414	-5.9499	-12.3812	-4.5039	-9.4560
$\theta_{3,0,0}$	10.7169	23.7481	7.2420	15.1088	5.4957	11.4995
$\theta_{4,0,0}$	-4.7944	-11.0137	-3.2305	-6.8804	-2.4466	-5.2185
$\theta_{0,1,0}$	44.026	82.066	22.411	43.843	15.248	30.049
$\theta_{1,1,0}$	-237.098	-559.081	-125.009	-299.888	-86.715	-209.389
$\theta_{2,1,0}$	705.899	1722.883	353.204	914.499	233.623	636.496
$\theta_{3,1,0}$	-852.779	-2300.592	-430.577	-1230.482	-281.745	-861.454
$\theta_{4,1,0}$	370.539	1067.711	188.684	573.944	122.676	403.538
$\theta_{0,2,0}$	458.05	937.48	243.44	434.66	151.87	263.02
$\theta_{0,3,0}$	-569.4	-4085.7	-1161.5	-2636.4	-843.0	-1730.9
$\theta_{0,1,1}$	-0.377	0.014	-0.413	-0.041	-0.397	-0.045
$\theta_{1,1,1}$	-4.906	-1.843	-3.105	-0.185	-2.442	0.186
$\theta_{2,1,1}$	17.922	23.522	7.352	4.744	4.819	0.253
$\theta_{3,1,1}$	-32.520	-63.725	-12.473	-20.859	-8.010	-9.674
$\theta_{4,1,1}$	16.604	37.596	5.760	12.971	3.483	6.457
$\theta_{0,2,1}$	3.30	51.23	14.71	35.04	13.60	24.82
$\theta_{0,3,1}$	1722.7	1352.0	423.0	264.2	154.4	69.5
$\bar{R}^2$	0.9992	0.9981	0.9995	0.9988	0.9996	0.9990
RMSE	0.0917	0.1235	0.0433	0.0548	0.0294	0.0357

Note: The response surface regression model is equation (10). The dependent variable is the simulated α-quantile of the test statistic. $\bar{R}^2$ denotes the adjusted coefficient of determination, and RMSE the root mean square error.

Table 21: Response surface estimates, F-statistic, case (iv)

	α = 1%		α = 5%		α = 10%	
	$I(0)$	$I(1)$	$I(0)$	$I(1)$	$I(0)$	$I(1)$
$\theta_{0,0,0}$	1.3614	2.5266	1.2800	2.4010	1.2237	2.3182
$\theta_{1,0,0}$	15.6352	15.4478	11.1327	10.3089	9.1079	7.9667
$\theta_{2,0,0}$	-19.2144	-24.2083	-13.4541	-16.0919	-10.8478	-12.2982
$\theta_{3,0,0}$	17.4958	25.7188	12.0425	16.8948	9.5992	12.8153
$\theta_{4,0,0}$	-6.9111	-11.0076	-4.7001	-7.1658	-3.7211	-5.4127
$\theta_{0,1,0}$	39.351	73.260	21.358	41.489	14.974	29.179
$\theta_{1,1,0}$	-153.902	-402.409	-91.075	-236.859	-67.028	-170.087
$\theta_{2,1,0}$	376.096	1053.617	230.114	654.846	167.876	477.117
$\theta_{3,1,0}$	-343.581	-1227.709	-235.226	-810.125	-174.785	-598.955
$\theta_{4,1,0}$	107.545	512.691	86.203	355.536	65.796	265.809
$\theta_{0,2,0}$	605.48	1043.38	317.08	499.07	203.89	311.82
$\theta_{0,3,0}$	-83.0	-3304.2	-1119.2	-2548.4	-926.5	-1816.6
$\theta_{0,1,1}$	-0.041	0.448	-0.265	0.143	-0.305	0.083
$\theta_{1,1,1}$	-13.289	-9.709	-7.844	-3.565	-6.091	-2.124
$\theta_{2,1,1}$	57.446	60.349	28.529	21.045	20.218	11.650
$\theta_{3,1,1}$	-92.826	-121.561	-42.514	-45.218	-28.516	-26.291
$\theta_{4,1,1}$	47.007	67.559	20.570	25.470	13.356	14.957
$\theta_{0,2,1}$	11.42	64.46	16.86	42.00	14.18	29.49
$\theta_{0,3,1}$	1837.1	1399.5	525.7	329.7	242.1	135.2
$\bar{R}^2$	0.9987	0.9967	0.9993	0.9981	0.9995	0.9984
RMSE	0.0816	0.1145	0.0381	0.0506	0.0264	0.0333

Note: The response surface regression model is equation (10). The dependent variable is the simulated α-quantile of the test statistic. $\bar{R}^2$ denotes the adjusted coefficient of determination, and RMSE the root mean square error.

Table 22: Response surface estimates, F-statistic, case (v)

	$\alpha = 1\%$		$\alpha = 5\%$		$\alpha = 10\%$	
	$I(0)$	$I(1)$	$I(0)$	$I(1)$	$I(0)$	$I(1)$
$\theta_{0,0,0}$	1.3230	2.4837	1.2588	2.3775	1.2077	2.3022
$\theta_{1,0,0}$	16.6413	17.8970	11.8758	12.3407	9.7073	9.7565
$\theta_{2,0,0}$	-6.7467	-13.7784	-4.6466	-9.1796	-3.6210	-6.8118
$\theta_{3,0,0}$	7.7086	16.6029	5.5895	11.1665	4.4870	8.2963
$\theta_{4,0,0}$	-3.2515	-7.4078	-2.4278	-5.0074	-1.9835	-3.7182
$\theta_{0,1,0}$	42.712	75.204	21.326	40.536	14.657	27.962
$\theta_{1,1,0}$	-166.956	-396.087	-96.367	-228.627	-72.165	-163.748
$\theta_{2,1,0}$	427.486	1081.487	243.336	645.539	178.211	463.610
$\theta_{3,1,0}$	-296.668	-1179.408	-204.692	-764.883	-162.466	-563.067
$\theta_{4,1,0}$	53.384	461.642	57.774	322.476	52.225	242.258
$\theta_{0,2,0}$	492.69	959.84	329.78	507.05	222.76	321.51
$\theta_{0,3,0}$	1789.0	-1775.4	-1224.8	-2686.6	-1172.6	-2048.6
$\theta_{0,1,1}$	0.160	0.605	-0.155	0.202	-0.226	0.108
$\theta_{1,1,1}$	-14.178	-10.448	-8.744	-4.124	-6.794	-2.399
$\theta_{2,1,1}$	56.969	63.238	28.919	21.946	20.278	10.891
$\theta_{3,1,1}$	-96.582	-131.545	-48.211	-51.837	-33.755	-29.984
$\theta_{4,1,1}$	49.063	72.154	23.735	28.658	16.316	16.757
$\theta_{0,2,1}$	-22.42	33.34	8.11	33.83	10.69	25.79
$\theta_{0,3,1}$	3272.7	2911.1	923.9	774.2	399.5	334.1
$\bar{R}^2$	0.9991	0.9986	0.9994	0.9991	0.9996	0.9993
RMSE	0.1414	0.1584	0.0730	0.0794	0.0518	0.0558

Note: The response surface regression model is equation (10). The dependent variable is the simulated α-quantile of the test statistic. $\bar{R}^2$ denotes the adjusted coefficient of determination, and RMSE the root mean square error.

Table 23: Response surface estimates, t-statistic, case (i)

	$\alpha = 1\%$		$\alpha = 5\%$		$\alpha = 10\%$	
	$I(0)$	$I(1)$	$I(0)$	$I(1)$	$I(0)$	$I(1)$
$\theta_{0,0,0}$	-2.5640	-7.3169	-1.9398	-6.6952	-1.6169	-6.3527
$\theta_{1,0,0}$	–	28.0719	–	28.0268	–	27.9488
$\theta_{2,0,0}$	–	-83.1077	–	-82.9353	–	-82.7398
$\theta_{3,0,0}$	–	113.4083	–	113.3623	–	113.1899
$\theta_{4,0,0}$	–	-53.6575	–	-53.7088	–	-53.6652
$\theta_{0,1,0}$	-8.304	-7.718	-1.813	15.466	0.836	24.494
$\theta_{1,1,0}$	58.111	-27.385	10.675	-216.221	-7.296	-288.605
$\theta_{2,1,0}$	-190.469	251.065	-34.105	862.473	24.671	1095.707
$\theta_{3,1,0}$	268.585	-471.426	47.185	-1335.576	-35.494	-1663.496
$\theta_{4,1,0}$	-128.897	256.894	-22.303	673.336	17.319	830.904
$\theta_{0,2,0}$	-77.82	-104.71	-17.93	19.25	-1.31	60.12
$\theta_{0,3,0}$	408.5	368.5	71.3	-462.5	-30.7	-769.2
$\theta_{0,1,1}$	0.141	1.526	0.088	1.654	0.103	1.761
$\theta_{1,1,1}$	-0.554	-9.560	0.139	-9.373	0.261	-9.517
$\theta_{2,1,1}$	1.248	31.426	-0.692	29.576	-1.169	29.520
$\theta_{3,1,1}$	0.206	-40.412	1.904	-38.664	2.305	-39.009
$\theta_{4,1,1}$	-0.573	18.108	-1.144	17.578	-1.279	17.913
$\theta_{0,2,1}$	-12.91	-38.58	-4.86	-26.41	-1.81	-22.92
$\theta_{0,3,1}$	38.2	269.6	10.0	226.4	-8.7	213.5
$\bar{R}^2$	0.9716	0.9987	0.9249	0.9993	0.7784	0.9993
RMSE	0.0164	0.0313	0.0077	0.0218	0.0060	0.0210

Note: The response surface regression model is equation (10). The dependent variable is the simulated α-quantile of the test statistic. $\bar{R}^2$ denotes the adjusted coefficient of determination, and RMSE the root mean square error.

Table 24: Response surface estimates, t-statistic, case (iii)

	$\alpha = 1\%$		$\alpha = 5\%$		$\alpha = 10\%$	
	$I(0)$	$I(1)$	$I(0)$	$I(1)$	$I(0)$	$I(1)$
$\theta_{0,0,0}$	-3.4345	-7.4681	-2.8642	-6.8423	-2.5692	-6.4989
$\theta_{1,0,0}$	–	26.6999	–	26.4474	–	26.2648
$\theta_{2,0,0}$	–	-81.3601	–	-80.5666	–	-80.1186
$\theta_{3,0,0}$	–	111.5262	–	110.4990	–	109.9530
$\theta_{4,0,0}$	–	-52.8701	–	-52.4117	–	-52.1717
$\theta_{0,1,0}$	-5.169	-4.975	3.978	17.930	7.895	27.135
$\theta_{1,1,0}$	6.222	-87.411	-48.810	-256.411	-73.151	-323.489
$\theta_{2,1,0}$	4.981	491.885	169.065	1008.331	244.286	1215.153
$\theta_{3,1,0}$	-27.267	-849.868	-245.128	-1551.623	-347.306	-1833.769
$\theta_{4,1,0}$	18.680	449.038	119.545	779.224	167.469	912.197
$\theta_{0,2,0}$	-132.33	-116.85	-51.83	24.99	-26.52	72.38
$\theta_{0,3,0}$	698.9	321.3	319.2	-552.9	178.4	-894.6
$\theta_{0,1,1}$	0.493	1.564	0.527	1.740	0.571	1.862
$\theta_{1,1,1}$	-0.271	-8.609	-0.261	-9.025	-0.236	-9.286
$\theta_{2,1,1}$	-3.465	27.142	-1.827	28.843	-1.595	29.721
$\theta_{3,1,1}$	10.250	-32.807	5.502	-37.386	4.401	-39.509
$\theta_{4,1,1}$	-6.340	14.024	-3.395	16.914	-2.643	18.232
$\theta_{0,2,1}$	-12.39	-39.97	-3.93	-28.44	-0.86	-25.33
$\theta_{0,3,1}$	-87.5	165.9	-60.4	183.6	-55.5	197.3
$\bar{R}^2$	0.9812	0.9977	0.9733	0.9986	0.9767	0.9986
RMSE	0.0211	0.0328	0.0109	0.0239	0.0086	0.0232

Note: The response surface regression model is equation (10). The dependent variable is the simulated α-quantile of the test statistic. $\bar{R}^2$ denotes the adjusted coefficient of determination, and RMSE the root mean square error.

Table 25: Response surface estimates, t-statistic, case (v)

	$\alpha = 1\%$		$\alpha = 5\%$		$\alpha = 10\%$	
	$I(0)$	$I(1)$	$I(0)$	$I(1)$	$I(0)$	$I(1)$
$\theta_{0,0,0}$	-3.9636	-7.6120	-3.4137	-6.9848	-3.1299	-6.6410
$\theta_{1,0,0}$	–	25.2386	–	24.8309	–	24.5586
$\theta_{2,0,0}$	–	-78.3522	–	-77.0823	–	-76.3657
$\theta_{3,0,0}$	–	108.2191	–	106.5084	–	105.5947
$\theta_{4,0,0}$	–	-51.4929	–	-50.6949	–	-50.2777
$\theta_{0,1,0}$	-1.976	-2.965	8.094	19.655	12.510	29.063
$\theta_{1,1,0}$	-47.416	-135.028	-96.698	-285.958	-120.306	-349.458
$\theta_{2,1,0}$	214.910	675.344	340.159	1109.102	406.410	1297.359
$\theta_{3,1,0}$	-354.745	-1135.951	-499.934	-1699.376	-583.357	-1948.866
$\theta_{4,1,0}$	185.122	595.013	245.932	852.345	283.037	967.675
$\theta_{0,2,0}$	-171.00	-129.71	-70.36	30.19	-35.26	86.35
$\theta_{0,3,0}$	815.8	208.9	424.6	-677.7	238.6	-1055.7
$\theta_{0,1,1}$	0.658	1.539	0.767	1.804	0.838	1.949
$\theta_{1,1,1}$	1.012	-6.526	-0.032	-8.204	-0.378	-8.747
$\theta_{2,1,1}$	-10.677	18.823	-4.107	26.704	-2.151	29.025
$\theta_{3,1,1}$	23.051	-19.457	10.038	-34.568	6.008	-39.147
$\theta_{4,1,1}$	-13.179	7.158	-5.906	15.612	-3.606	18.214
$\theta_{0,2,1}$	-15.77	-44.36	-5.61	-32.39	-1.55	-28.80
$\theta_{0,3,1}$	-164.2	95.8	-108.0	158.9	-96.4	184.5
$\bar{R}^2$	0.9836	0.9966	0.9777	0.9976	0.9796	0.9976
RMSE	0.0261	0.0350	0.0149	0.0263	0.0121	0.0255

Note: The response surface regression model is equation (10). The dependent variable is the simulated α-quantile of the test statistic. $\bar{R}^2$ denotes the adjusted coefficient of determination, and RMSE the root mean square error.