Bias-corrected method of moments estimators for dynamic panel data models

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Supplementary Appendix

Appendix C Monte Carlo simulation

C.1 Details of the data-generating process

Following Kiviet et al. (2017), we hold fixed the fraction of the variance of x_{it} that is due to the individual-specific effects (EVF_x) and the variance fraction of $\pi_\mu \mu_i$ in the composite individual-specific effects (IEF_x^μ). This allows us to endogenously determine

$$\begin{aligned} \pi_\mu &= (1-\gamma)\sqrt{EVF_x IEF_x^\mu}, \\ \pi_\lambda &= (1-\gamma)\sqrt{EVF_x (1 - IEF_x^\mu)}, \\ \sigma_\epsilon^2 &= \sqrt{(1-\gamma^2)(1-EVF_x)}. \end{aligned}$$

With the normalization $\sigma_u^2 = 1$, and by further fixing the direct cumulated effect of μ_i on y_{it} relative to the noise (DEN_y^η) and the signal-to-noise ratio (SNR), we obtain

$$\begin{aligned} \sigma_\mu &= (1-\alpha) DEN_y^\eta, \\ \beta &= \sqrt{\frac{(1-\alpha\gamma)(SNR - \alpha^2(1+SNR))}{(1+\alpha\gamma)(1-EVF_x)}}. \end{aligned}$$

For details, see Kiviet et al. (2017, Section 4). Fixing $\gamma = 0.4$, $EVF_x = IEF_x^\mu = 0.3$, $DEN_y^\eta = 4$, and $SNR = 5$ implies $\beta \approx 2.044$ under moderate persistence and $\beta \approx 0.307$

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in the high-persistence case.

C.2 Construction of the estimators

Besides the within-groups estimator (WG) and our bias-corrected estimator (BC), we consider three generalized method of moments (GMM) estimators:

- The one-step Arellano and Bond (1991) GMM estimator (AB-GMM) utilizes the moment conditions[1]

$$E[y_{i,t-s}\Delta e_{it}(\boldsymbol{\theta}_0)] = 0, \quad 2 \leq s \leq 4, \quad t = s, \ldots, T,$$
$$E\left[\sum_{t=2}^{T} \Delta x_{it}\Delta e_{it}(\boldsymbol{\theta}_0)\right] = 0.$$

  With a weighting matrix that is optimal under homoskedasticity and absence of serial correlation in the idiosyncratic error component, the one-step GMM estimator equals the two-stage least squares estimator.

- The two-step Ahn and Schmidt (1995) GMM estimator (AS-GMM) utilizes the nonlinear moment conditions $E[e_{iT}(\boldsymbol{\theta}_0)\Delta e_{it}(\boldsymbol{\theta}_0)] = 0$, $t = 2, \ldots, T-1$, in addition to the moment conditions of the AB-GMM estimator and an intercept. The optimal weighting matrix is computed based on the residuals from an inefficient estimator with block-diagonal weighting matrix. The block corresponding to the AB-GMM moment conditions is identical to the AB-GMM weighting matrix. The block corresponding to the nonlinear moment conditions is an identity matrix.

- The two-step Blundell and Bond (1998) GMM estimator (BB-GMM) utilizes the moment conditions $E[\Delta y_{i,t-1}e_{it}(\boldsymbol{\theta}_0)] = 0$, $t = 2, \ldots, T$, in addition to the moment conditions of the AB-GMM estimator and an intercept. The optimal weighting matrix is computed based on the residuals from the inefficient two-stage least squares estimator.

We use the same moment conditions under AR(1) and AR(3) dynamics. In the simulations with AR(1) dynamics, we further consider the Hsiao et al. (2002) quasi-maximum likelihood (QML) estimator with the initial-observations parameterization

$$E[\Delta y_{i1}|\Delta x_{i1}, \ldots, \Delta x_{iT}] = b + \sum_{s=1}^{3} \pi_s \Delta x_{is}.$$

[1]We do not use all available lags of the dependent variable to avoid instrument proliferation. As demonstrated by Hayakawa et al. (2019), 3 lags are sufficient to achieve a reasonable degree of efficiency.

C.3 Estimation of the variance-covariance matrix

We consider the following two estimators for the variance-covariance matrix:

$$\mathbf{V}_{NT}(\widehat{\boldsymbol{\theta}}) = \frac{1}{N}\mathbf{D}_{NT}(\widehat{\boldsymbol{\theta}})^{-1}\mathbf{S}_{NT}(\widehat{\boldsymbol{\theta}})\mathbf{D}_{NT}(\widehat{\boldsymbol{\theta}})'^{-1},$$
$$\mathbf{V}_{NT}^{csd}(\widehat{\boldsymbol{\theta}}) = \frac{1}{NT}\mathbf{D}_{NT}(\widehat{\boldsymbol{\theta}})^{-1}\mathbf{S}_{NT}^{csd}(\widehat{\boldsymbol{\theta}})\mathbf{D}_{NT}(\widehat{\boldsymbol{\theta}})'^{-1}.$$

Variance-covariance estimator $\mathbf{V}_{NT}(\widehat{\boldsymbol{\theta}})$ is the conventional fixed-T cluster-robust estimator clustered at the individual level. Estimator $\mathbf{V}_{NT}^{csd}(\widehat{\boldsymbol{\theta}})$ is a robust large-T estimator clustered at the time periods. For the WG estimator, we have

$$\mathbf{D}_{NT}(\widehat{\boldsymbol{\theta}}) = \frac{1}{N}\sum_{i=1}^{N}\boldsymbol{\nabla}\mathbf{g}_{Ti}(\widehat{\boldsymbol{\theta}}), \quad \mathbf{S}_{NT}(\widehat{\boldsymbol{\theta}}) = \frac{1}{N}\sum_{i=1}^{N}\mathbf{g}_{Ti}(\widehat{\boldsymbol{\theta}})\mathbf{g}_{Ti}(\widehat{\boldsymbol{\theta}})',$$
$$\mathbf{S}_{NT}^{csd}(\widehat{\boldsymbol{\theta}}) = \frac{1}{NT}\sum_{t=1}^{T}\mathbf{W}_t'[\mathbf{e}_t(\widehat{\boldsymbol{\theta}}) - \overline{\mathbf{e}}(\widehat{\boldsymbol{\theta}})][\mathbf{e}_t(\widehat{\boldsymbol{\theta}}) - \overline{\mathbf{e}}(\widehat{\boldsymbol{\theta}})]'\mathbf{W}_t.$$

Similarly, for the BC estimator we compute

$$\mathbf{D}_{NT}(\widehat{\boldsymbol{\theta}}) = \frac{1}{N}\sum_{i=1}^{N}\boldsymbol{\nabla}_{\theta}\mathbf{m}_{Ti}(\widehat{\boldsymbol{\theta}}), \quad \mathbf{S}_{NT}(\widehat{\boldsymbol{\theta}}) = \frac{1}{N}\sum_{i=1}^{N}\mathbf{m}_{Ti}(\widehat{\boldsymbol{\theta}})\mathbf{m}_{Ti}(\widehat{\boldsymbol{\theta}})',$$
$$\mathbf{S}_{NT}^{csd}(\widehat{\boldsymbol{\theta}}) = \frac{1}{NT}\sum_{t=1}^{T}\mathbf{Z}_t(\widehat{\boldsymbol{\theta}})'[\mathbf{e}_t(\widehat{\boldsymbol{\theta}}) - \overline{\mathbf{e}}(\widehat{\boldsymbol{\theta}})][\mathbf{e}_t(\widehat{\boldsymbol{\theta}}) - \overline{\mathbf{e}}(\widehat{\boldsymbol{\theta}})]'\mathbf{Z}_t(\widehat{\boldsymbol{\theta}}).$$

For the GMM estimators with moment functions $\tilde{\mathbf{m}}_{Ti}(\boldsymbol{\theta})$ and weighting matrix $\mathbf{W}_{NT}$, we get

$$\mathbf{D}_{NT}(\widehat{\boldsymbol{\theta}}) = \left(\frac{1}{N}\sum_{i=1}^{N}\boldsymbol{\nabla}\tilde{\mathbf{m}}_{Ti}(\widehat{\boldsymbol{\theta}})'\right)\mathbf{W}_{NT}\left(\frac{1}{N}\sum_{i=1}^{N}\boldsymbol{\nabla}\tilde{\mathbf{m}}_{Ti}(\widehat{\boldsymbol{\theta}})\right),$$
$$\mathbf{S}_{NT}(\widehat{\boldsymbol{\theta}}) = \left(\frac{1}{N}\sum_{i=1}^{N}\boldsymbol{\nabla}\tilde{\mathbf{m}}_{Ti}(\widehat{\boldsymbol{\theta}})'\right)\mathbf{W}_{NT}\left(\frac{1}{N}\sum_{i=1}^{N}\tilde{\mathbf{m}}_{Ti}(\widehat{\boldsymbol{\theta}})\tilde{\mathbf{m}}_{Ti}(\widehat{\boldsymbol{\theta}})'\right)\mathbf{W}_{NT}\left(\frac{1}{N}\sum_{i=1}^{N}\boldsymbol{\nabla}\tilde{\mathbf{m}}_{Ti}(\widehat{\boldsymbol{\theta}})\right).$$

For the two-step AS-GMM and BB-GMM estimators with optimal weighting matrix, the variance matrix is estimated as $\mathbf{V}_{NT}(\widehat{\boldsymbol{\theta}}) = N^{-1}\mathbf{D}_{NT}(\widehat{\boldsymbol{\theta}})^{-1}$, and subsequently the Windmeijer (2005) finite-sample correction is applied. For the QML estimator, $\mathbf{D}_{NT}(\widehat{\boldsymbol{\theta}})$ is the negative Hessian matrix and $\mathbf{S}_{NT}(\widehat{\boldsymbol{\theta}})$ the outer product of the gradient.

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Table 1: Simulation results: baseline model (IID), $N = 50$

	α						β					
	WG	BC	AB-GMM	AS-GMM	BB-GMM	QML	WG	BC	AB-GMM	AS-GMM	BB-GMM	QML
	$\alpha = 0.4$											
$T = 5$												
Bias	-0.077	0.001	-0.038	0.067	0.097	-0.003	-0.005	-0.001	-0.022	0.019	-0.025	-0.025
RMSE	0.086	0.041	0.117	0.150	0.140	0.041	0.093	0.093	0.134	0.148	0.142	0.094
$T = 10$												
Bias	-0.034	0.000	-0.020	0.109	0.129	-0.006	0.015	0.001	-0.008	0.038	-0.042	-0.018
RMSE	0.041	0.023	0.055	0.145	0.143	0.023	0.062	0.060	0.075	0.117	0.103	0.062
$T = 25$												
Bias	-0.014	-0.001	-0.008	0.094	0.142	-0.004	0.013	0.003	0.008	0.025	-0.038	-0.006
RMSE	0.019	0.013	0.023	0.108	0.147	0.013	0.037	0.035	0.044	0.080	0.067	0.035
$T = 50$												
Bias	-0.006	0.000	-0.005	0.078	0.134	-0.002	0.006	0.000	0.006	0.015	-0.043	-0.004
RMSE	0.011	0.009	0.014	0.087	0.138	0.009	0.026	0.025	0.031	0.058	0.059	0.025
	$\alpha = 0.9$											
$T = 5$												
Bias	-0.433	-0.034	-0.442	-0.044	0.018	-0.003	-0.048	-0.001	-0.058	-0.005	-0.008	-0.033
RMSE	0.438	0.124	0.561	0.205	0.091	0.156	0.102	0.095	0.124	0.125	0.122	0.094
$T = 10$												
Bias	-0.223	-0.004	-0.249	0.007	0.031	-0.022	-0.016	0.002	-0.031	0.002	-0.009	-0.039
RMSE	0.226	0.067	0.321	0.084	0.053	0.073	0.064	0.063	0.086	0.090	0.086	0.071
$T = 25$												
Bias	-0.085	0.000	-0.078	0.009	0.027	-0.018	0.004	0.003	0.002	0.013	0.003	-0.024
RMSE	0.087	0.025	0.103	0.046	0.037	0.026	0.034	0.033	0.046	0.055	0.049	0.041
$T = 50$												
Bias	-0.039	-0.001	-0.033	0.006	0.023	-0.009	0.004	0.001	0.005	0.007	-0.002	-0.017
RMSE	0.041	0.012	0.044	0.033	0.035	0.014	0.025	0.024	0.033	0.041	0.035	0.029

Note: The comparison includes the within-groups estimator (WG), the bias-corrected method of moments estimator (BC), the one-step Arellano and Bond (1991) GMM estimator (AB-GMM), the two-step Ahn and Schmidt (1995) GMM estimator (AS-GMM), the two-step Blundell and Bond (1998) GMM estimator (BB-GMM), and the Hsiao et al. (2002) QML estimator. Reported are the average bias of the estimates and the root mean square error (RMSE).

Table 2: Simulation results: baseline model (IID), $N = 200$

	α						β					
	WG	BC	AB-GMM	AS-GMM	BB-GMM	QML	WG	BC	AB-GMM	AS-GMM	BB-GMM	QML
	$\alpha = 0.4$											
$T = 5$												
Bias	-0.079	-0.001	-0.008	0.026	0.024	-0.004	-0.002	0.000	-0.004	0.011	-0.002	-0.023
RMSE	0.081	0.021	0.058	0.065	0.054	0.021	0.046	0.046	0.063	0.066	0.060	0.051
$T = 10$												
Bias	-0.035	-0.001	-0.005	0.017	0.022	-0.007	0.014	0.000	-0.002	-0.001	-0.012	-0.020
RMSE	0.037	0.011	0.025	0.030	0.034	0.013	0.033	0.030	0.038	0.041	0.042	0.036
$T = 25$												
Bias	-0.013	0.000	-0.002	0.022	0.036	-0.004	0.011	0.001	0.002	0.001	-0.014	-0.008
RMSE	0.015	0.007	0.011	0.027	0.039	0.008	0.021	0.018	0.022	0.028	0.030	0.019
$T = 50$												
Bias	-0.006	0.000	-0.001	0.032	0.060	-0.002	0.005	0.000	0.001	0.005	-0.013	-0.005
RMSE	0.008	0.004	0.007	0.035	0.061	0.005	0.014	0.013	0.016	0.025	0.023	0.014
	$\alpha = 0.9$											
$T = 5$												
Bias	-0.430	-0.006	-0.191	0.000	0.001	-0.002	-0.048	0.000	-0.025	-0.002	-0.004	-0.033
RMSE	0.432	0.082	0.306	0.122	0.058	0.102	0.066	0.047	0.068	0.060	0.059	0.055
$T = 10$												
Bias	-0.221	0.004	-0.083	-0.001	0.008	-0.029	-0.016	0.000	-0.010	-0.004	-0.007	-0.038
RMSE	0.222	0.044	0.124	0.060	0.033	0.040	0.034	0.030	0.041	0.042	0.042	0.047
$T = 25$												
Bias	-0.084	0.000	-0.020	0.006	0.015	-0.016	0.002	0.001	0.001	0.001	-0.003	-0.026
RMSE	0.084	0.012	0.035	0.026	0.022	0.019	0.018	0.017	0.023	0.027	0.026	0.031
$T = 50$												
Bias	-0.039	0.000	-0.009	0.010	0.022	-0.009	0.003	0.000	0.001	0.003	0.000	-0.018
RMSE	0.040	0.006	0.017	0.019	0.024	0.010	0.012	0.012	0.017	0.019	0.018	0.022

Note: The comparison includes the within-groups estimator (WG), the bias-corrected method of moments estimator (BC), the one-step Arellano and Bond (1991) GMM estimator (AB-GMM), the two-step Ahn and Schmidt (1995) GMM estimator (AS-GMM), the two-step Blundell and Bond (1998) GMM estimator (BB-GMM), and the Hsiao et al. (2002) QML estimator. Reported are the average bias of the estimates and the root mean square error (RMSE).

Table 3: Simulation results: higher-order dynamics, $N = 50$

	α					β				
	WG	BC	AB-GMM	AS-GMM	BB-GMM	WG	BC	AB-GMM	AS-GMM	BB-GMM
	$\alpha = \alpha_1 + \alpha_2 + \alpha_3 = 0.4$									
$T = 5$										
Bias	-0.143	0.000	-0.113	0.103	0.222	-0.038	-0.002	-0.062	0.036	0.026
RMSE	0.156	0.069	0.253	0.215	0.258	0.103	0.098	0.179	0.165	0.143
$T = 10$										
Bias	-0.056	0.001	-0.043	0.199	0.254	0.006	0.003	-0.017	0.064	0.012
RMSE	0.065	0.033	0.102	0.231	0.266	0.062	0.062	0.090	0.126	0.095
$T = 25$										
Bias	-0.020	0.000	-0.018	0.192	0.248	0.007	0.000	-0.002	0.050	0.013
RMSE	0.026	0.016	0.044	0.204	0.253	0.037	0.037	0.048	0.084	0.057
$T = 50$										
Bias	-0.009	0.000	-0.017	0.169	0.219	0.004	0.000	0.001	0.040	0.008
RMSE	0.015	0.011	0.031	0.179	0.225	0.026	0.025	0.033	0.063	0.042
	$\alpha = \alpha_1 + \alpha_2 + \alpha_3 = 0.9$									
$T = 5$										
Bias	-0.558	0.012	-0.588	-0.093	0.035	-0.063	-0.003	-0.074	-0.014	-0.006
RMSE	0.566	0.174	0.719	0.290	0.100	0.108	0.098	0.132	0.130	0.129
$T = 10$										
Bias	-0.282	0.019	-0.341	0.006	0.047	-0.024	0.004	-0.038	-0.004	-0.006
RMSE	0.287	0.093	0.426	0.094	0.067	0.064	0.061	0.089	0.090	0.087
$T = 25$										
Bias	-0.105	0.002	-0.135	0.016	0.031	-0.003	0.001	-0.009	0.007	0.000
RMSE	0.107	0.034	0.171	0.056	0.048	0.036	0.035	0.050	0.055	0.051
$T = 50$										
Bias	-0.048	0.000	-0.080	-0.003	0.014	0.000	-0.001	-0.001	0.007	-0.003
RMSE	0.050	0.014	0.096	0.057	0.053	0.024	0.024	0.034	0.041	0.036

Note: Note: The comparison includes the within-groups estimator (WG), the bias-corrected method of moments estimator (BC), the one-step Arellano and Bond (1991) GMM estimator (AB-GMM), the two-step Ahn and Schmidt (1995) GMM estimator (AS-GMM), and the two-step Blundell and Bond (1998) GMM estimator (BB-GMM). Reported are the average bias of the estimates and the root mean square error (RMSE).

Table 4: Simulation results: higher-order dynamics, $N = 200$

	α					β				
	WG	BC	AB-GMM	AS-GMM	BB-GMM	WG	BC	AB-GMM	AS-GMM	BB-GMM
	$\alpha = \alpha_1 + \alpha_2 + \alpha_3 = 0.4$									
$T = 5$										
Bias	-0.141	0.000	-0.038	0.069	0.085	-0.035	0.000	-0.019	0.033	0.019
RMSE	0.144	0.034	0.128	0.136	0.125	0.059	0.049	0.088	0.091	0.072
$T = 10$										
Bias	-0.057	0.000	-0.012	0.059	0.085	0.003	0.000	-0.005	0.018	0.007
RMSE	0.059	0.016	0.051	0.087	0.102	0.030	0.030	0.042	0.053	0.044
$T = 25$										
Bias	-0.020	0.000	-0.006	0.074	0.114	0.006	-0.001	-0.001	0.017	0.010
RMSE	0.022	0.008	0.023	0.082	0.119	0.019	0.018	0.023	0.033	0.028
$T = 50$										
Bias	-0.010	0.000	-0.004	0.119	0.169	0.005	0.000	0.000	0.034	0.028
RMSE	0.011	0.005	0.014	0.123	0.171	0.014	0.013	0.017	0.041	0.034
	$\alpha = \alpha_1 + \alpha_2 + \alpha_3 = 0.9$									
$T = 5$										
Bias	-0.554	0.032	-0.309	-0.026	0.022	-0.063	0.003	-0.036	-0.004	-0.001
RMSE	0.556	0.125	0.444	0.157	0.081	0.077	0.050	0.076	0.063	0.063
$T = 10$										
Bias	-0.281	0.028	-0.132	0.004	0.026	-0.025	0.002	-0.015	-0.003	-0.005
RMSE	0.282	0.078	0.192	0.077	0.053	0.039	0.030	0.043	0.042	0.042
$T = 25$										
Bias	-0.104	0.001	-0.042	0.012	0.032	-0.004	-0.001	-0.003	0.000	-0.002
RMSE	0.105	0.016	0.063	0.037	0.041	0.018	0.017	0.024	0.027	0.026
$T = 50$										
Bias	-0.048	0.000	-0.023	0.024	0.041	0.001	0.000	0.000	0.004	0.002
RMSE	0.048	0.007	0.034	0.034	0.043	0.013	0.012	0.018	0.020	0.018

Note: The comparison includes the within-groups estimator (WG), the bias-corrected method of moments estimator (BC), the one-step Arellano and Bond (1991) GMM estimator (AB-GMM), the two-step Ahn and Schmidt (1995) GMM estimator (AS-GMM), and the two-step Blundell and Bond (1998) GMM estimator (BB-GMM). Reported are the average bias of the estimates and the root mean square error (RMSE).

Table 5: Simulation results: heteroskedasticity, $N = 50$

	α						β					
	WG	BC	AB-GMM	AS-GMM	BB-GMM	QML	WG	BC	AB-GMM	AS-GMM	BB-GMM	QML
	$\alpha = 0.4$											
$T = 5$												
Bias	-0.075	0.001	-0.032	0.041	0.092	-0.004	-0.001	0.002	-0.018	0.014	-0.008	-0.024
RMSE	0.095	0.053	0.116	0.116	0.127	0.047	0.097	0.098	0.133	0.132	0.125	0.100
$T = 10$												
Bias	-0.034	0.000	-0.017	0.079	0.123	-0.008	0.018	0.004	0.001	0.040	-0.023	-0.019
RMSE	0.042	0.024	0.054	0.128	0.138	0.024	0.066	0.062	0.079	0.123	0.088	0.065
$T = 25$												
Bias	-0.013	0.000	-0.008	0.087	0.147	-0.005	0.009	-0.001	0.002	0.027	-0.035	-0.011
RMSE	0.019	0.014	0.024	0.114	0.152	0.015	0.038	0.037	0.046	0.096	0.065	0.039
$T = 50$												
Bias	-0.007	-0.001	-0.006	0.081	0.145	-0.003	0.006	0.000	0.006	0.017	-0.032	-0.005
RMSE	0.012	0.010	0.016	0.096	0.149	0.010	0.027	0.026	0.032	0.068	0.050	0.027
	$\alpha = 0.9$											
$T = 5$												
Bias	-0.405	-0.100	-0.373	-0.096	0.017	-0.058	-0.047	-0.012	-0.054	-0.012	0.006	-0.033
RMSE	0.437	0.249	0.536	0.272	0.066	0.252	0.105	0.096	0.132	0.112	0.102	0.098
$T = 10$												
Bias	-0.217	-0.028	-0.244	-0.017	0.024	-0.041	-0.018	-0.004	-0.034	0.003	0.007	-0.041
RMSE	0.230	0.107	0.307	0.113	0.043	0.103	0.063	0.060	0.085	0.079	0.070	0.072
$T = 25$												
Bias	-0.084	0.000	-0.103	-0.002	0.025	-0.018	-0.001	-0.002	-0.011	0.007	0.009	-0.030
RMSE	0.088	0.033	0.136	0.076	0.032	0.030	0.036	0.035	0.050	0.067	0.044	0.047
$T = 50$												
Bias	-0.040	-0.001	-0.049	0.001	0.026	-0.010	0.004	0.001	0.004	0.011	0.014	-0.018
RMSE	0.042	0.017	0.064	0.059	0.030	0.017	0.026	0.025	0.034	0.052	0.034	0.032

Note: The comparison includes the within-groups estimator (WG), the bias-corrected method of moments estimator (BC), the one-step Arellano and Bond (1991) GMM estimator (AB-GMM), the two-step Ahn and Schmidt (1995) GMM estimator (AS-GMM), the two-step Blundell and Bond (1998) GMM estimator (BB-GMM), and the Hsiao et al. (2002) QML estimator. Reported are the average bias of the estimates and the root mean square error (RMSE).

Table 6: Simulation results: heteroskedasticity, $N = 200$

	α						β					
	WG	BC	AB-GMM	AS-GMM	BB-GMM	QML	WG	BC	AB-GMM	AS-GMM	BB-GMM	QML
	$\alpha = 0.4$											
$T = 5$												
Bias	-0.071	0.003	-0.011	0.010	0.018	-0.002	-0.002	0.001	-0.003	0.005	0.000	-0.025
RMSE	0.084	0.037	0.056	0.040	0.044	0.028	0.048	0.047	0.063	0.055	0.054	0.053
$T = 10$												
Bias	-0.032	0.001	-0.006	0.012	0.019	-0.007	0.014	0.000	-0.003	0.000	-0.012	-0.022
RMSE	0.037	0.013	0.025	0.026	0.027	0.014	0.034	0.030	0.039	0.037	0.039	0.039
$T = 25$												
Bias	-0.013	0.000	-0.002	0.018	0.033	-0.005	0.010	0.000	0.001	0.004	-0.010	-0.011
RMSE	0.015	0.007	0.012	0.026	0.036	0.009	0.021	0.018	0.022	0.024	0.024	0.022
$T = 50$												
Bias	-0.006	0.000	-0.001	0.029	0.057	-0.003	0.006	0.000	0.001	0.005	-0.010	-0.006
RMSE	0.008	0.005	0.007	0.035	0.059	0.006	0.014	0.013	0.016	0.028	0.020	0.014
	$\alpha = 0.9$											
$T = 5$												
Bias	-0.407	-0.092	-0.198	-0.050	0.005	-0.077	-0.048	-0.012	-0.028	-0.012	-0.003	-0.037
RMSE	0.435	0.244	0.326	0.187	0.038	0.238	0.070	0.057	0.073	0.066	0.059	0.062
$T = 10$												
Bias	-0.215	-0.022	-0.097	-0.006	0.007	-0.040	-0.015	-0.001	-0.013	-0.002	-0.001	-0.037
RMSE	0.225	0.097	0.148	0.038	0.018	0.087	0.035	0.031	0.044	0.036	0.035	0.048
$T = 25$												
Bias	-0.083	0.002	-0.029	-0.001	0.009	-0.017	0.002	0.000	-0.001	0.001	0.002	-0.027
RMSE	0.085	0.026	0.046	0.035	0.014	0.023	0.019	0.018	0.024	0.021	0.020	0.033
$T = 50$												
Bias	-0.039	0.000	-0.013	0.003	0.011	-0.010	0.004	0.001	0.002	0.005	0.007	-0.018
RMSE	0.040	0.009	0.022	0.029	0.014	0.012	0.013	0.012	0.017	0.021	0.018	0.022

Note: Note: The comparison includes the within-groups estimator (WG), the bias-corrected method of moments estimator (BC), the one-step Arellano and Bond (1991) GMM estimator (AB-GMM), the two-step Ahn and Schmidt (1995) GMM estimator (AS-GMM), the two-step Blundell and Bond (1998) GMM estimator (BB-GMM), and the Hsiao et al. (2002) QML estimator. Reported are the average bias of the estimates and the root mean square error (RMSE).

Table 7: Simulation results: uniform cross-sectional dependence, $N = 50$

	α						β					
	WG	BC	AB-GMM	AS-GMM	BB-GMM	QML	WG	BC	AB-GMM	AS-GMM	BB-GMM	QML
	$\alpha = 0.4$											
$T = 10$												
Bias	-0.037	-0.004	-0.070	-0.015	0.120	-0.015	0.014	0.001	-0.018	-0.031	-0.057	-0.032
RMSE	0.065	0.055	0.135	0.182	0.143	0.054	0.068	0.067	0.106	0.180	0.122	0.078
Size	0.584	0.415	0.501	0.492	0.578	0.449	0.086	0.412	0.154	0.189	0.111	0.121
rob-Size	0.186	0.065	n.a.	n.a.	n.a.	n.a.	0.099	0.065	n.a.	n.a.	n.a.	n.a.
$T = 25$												
Bias	-0.015	-0.002	-0.033	-0.001	0.135	-0.008	0.012	0.002	0.018	0.013	-0.035	-0.013
RMSE	0.036	0.033	0.062	0.137	0.143	0.033	0.045	0.043	0.057	0.127	0.071	0.046
Size	0.564	0.478	0.551	0.530	0.909	0.511	0.105	0.477	0.127	0.289	0.101	0.123
rob-Size	0.101	0.066	n.a.	n.a.	n.a.	n.a.	0.078	0.066	n.a.	n.a.	n.a.	n.a.
$T = 50$												
Bias	-0.007	-0.001	-0.025	-0.009	0.136	-0.005	0.007	0.001	0.025	0.012	-0.040	-0.006
RMSE	0.024	0.023	0.042	0.123	0.141	0.023	0.032	0.032	0.044	0.106	0.060	0.033
Size	0.540	0.496	0.568	0.535	0.961	0.518	0.139	0.496	0.173	0.298	0.143	0.147
rob-Size	0.071	0.054	n.a.	n.a.	n.a.	n.a.	0.071	0.054	n.a.	n.a.	n.a.	n.a.
	$\alpha = 0.9$											
$T = 10$												
Bias	-0.266	-0.092	-0.492	-0.292	-0.085	-0.080	-0.020	-0.006	-0.056	-0.029	0.016	-0.089
RMSE	0.313	0.197	0.582	0.479	0.146	0.223	0.063	0.061	0.107	0.155	0.101	0.115
Size	0.949	0.527	0.924	0.807	0.503	0.865	0.082	0.517	0.271	0.149	0.071	0.432
rob-Size	0.508	0.078	n.a.	n.a.	n.a.	n.a.	0.153	0.085	n.a.	n.a.	n.a.	n.a.
$T = 25$												
Bias	-0.111	-0.032	-0.303	-0.163	-0.038	-0.050	0.002	0.002	-0.006	0.008	0.024	-0.061
RMSE	0.137	0.094	0.352	0.332	0.072	0.100	0.036	0.036	0.063	0.111	0.065	0.090
Size	0.929	0.594	0.945	0.847	0.569	0.796	0.064	0.539	0.190	0.227	0.114	0.342
rob-Size	0.342	0.039	n.a.	n.a.	n.a.	n.a.	0.101	0.048	n.a.	n.a.	n.a.	n.a.
$T = 50$												
Bias	-0.056	-0.017	-0.217	-0.117	-0.016	-0.033	0.005	0.002	0.024	0.023	0.017	-0.037
RMSE	0.075	0.057	0.246	0.276	0.044	0.058	0.025	0.024	0.049	0.100	0.045	0.053
Size	0.898	0.713	0.972	0.829	0.429	0.818	0.065	0.614	0.218	0.306	0.105	0.324
rob-Size	0.224	0.040	n.a.	n.a.	n.a.	n.a.	0.059	0.045	n.a.	n.a.	n.a.	n.a.

Note: The comparison includes the within-groups estimator (WG), the bias-corrected method of moments estimator (BC), the one-step Arellano and Bond (1991) GMM estimator (AB-GMM), the two-step Ahn and Schmidt (1995) GMM estimator (AS-GMM), the two-step Blundell and Bond (1998) GMM estimator (BB-GMM), and the Hsiao et al. (2002) QML estimator. Reported are the average bias of the estimates, the root mean square error (RMSE), and the empirical size of the Wald statistics for the hypothesis $\beta = \beta_0$. 'rob-Size' refers to the Wald test employing robust standard errors considered in Theorem 2.

Table 8: Simulation results: uniform cross-sectional dependence, $N = 200$

	α						β					
	WG	BC	AB-GMM	AS-GMM	BB-GMM	QML	WG	BC	AB-GMM	AS-GMM	BB-GMM	QML
	$\alpha = 0.4$											
$T = 10$												
Bias	-0.037	-0.003	-0.064	-0.053	0.024	-0.014	0.016	0.002	-0.015	-0.066	-0.089	-0.031
RMSE	0.061	0.049	0.122	0.199	0.066	0.048	0.040	0.036	0.078	0.132	0.112	0.052
Size	0.775	0.670	0.686	0.712	0.318	0.691	0.138	0.670	0.319	0.365	0.376	0.246
rob-Size	0.175	0.061	n.a.	n.a.	n.a.	n.a.	0.131	0.061	n.a.	n.a.	n.a.	n.a.
$T = 25$												
Bias	-0.014	-0.001	-0.030	-0.038	0.044	-0.008	0.011	0.001	0.017	-0.010	-0.037	-0.014
RMSE	0.034	0.031	0.060	0.158	0.060	0.031	0.032	0.030	0.041	0.083	0.053	0.034
Size	0.733	0.689	0.759	0.796	0.631	0.705	0.282	0.686	0.270	0.304	0.198	0.280
rob-Size	0.103	0.043	n.a.	n.a.	n.a.	n.a.	0.092	0.043	n.a.	n.a.	n.a.	n.a.
$T = 50$												
Bias	-0.006	0.000	-0.020	-0.025	0.050	-0.003	0.004	-0.001	0.022	0.017	-0.009	-0.009
RMSE	0.022	0.022	0.038	0.132	0.058	0.021	0.023	0.022	0.033	0.074	0.026	0.024
Size	0.709	0.688	0.761	0.799	0.827	0.688	0.271	0.688	0.368	0.523	0.169	0.305
rob-Size	0.060	0.055	n.a.	n.a.	n.a.	n.a.	0.061	0.055	n.a.	n.a.	n.a.	n.a.
	$\alpha = 0.9$											
$T = 10$												
Bias	-0.267	-0.092	-0.516	-0.365	-0.179	-0.085	-0.018	-0.005	-0.057	-0.045	0.005	-0.087
RMSE	0.311	0.191	0.605	0.558	0.244	0.216	0.036	0.032	0.087	0.111	0.060	0.104
Size	0.982	0.614	0.974	0.919	0.873	0.917	0.116	0.594	0.497	0.202	0.027	0.691
rob-Size	0.524	0.083	n.a.	n.a.	n.a.	n.a.	0.188	0.083	n.a.	n.a.	n.a.	n.a.
$T = 25$												
Bias	-0.112	-0.034	-0.321	-0.190	-0.092	-0.052	0.002	0.001	-0.009	0.002	0.013	-0.063
RMSE	0.138	0.094	0.370	0.384	0.129	0.100	0.018	0.018	0.044	0.074	0.038	0.087
Size	0.959	0.693	0.973	0.931	0.904	0.903	0.057	0.613	0.328	0.223	0.078	0.664
rob-Size	0.356	0.043	n.a.	n.a.	n.a.	n.a.	0.090	0.051	n.a.	n.a.	n.a.	n.a.
$T = 50$												
Bias	-0.052	-0.012	-0.206	-0.130	-0.044	-0.029	0.004	0.001	0.024	0.023	0.010	-0.037
RMSE	0.071	0.054	0.235	0.311	0.073	0.053	0.014	0.013	0.037	0.062	0.027	0.046
Size	0.946	0.809	0.991	0.942	0.899	0.899	0.081	0.715	0.434	0.486	0.217	0.684
rob-Size	0.210	0.022	n.a.	n.a.	n.a.	n.a.	0.073	0.028	n.a.	n.a.	n.a.	n.a.

Note: The comparison includes the within-groups estimator (WG), the bias-corrected method of moments estimator (BC), the one-step Arellano and Bond (1991) GMM estimator (AB-GMM), the two-step Ahn and Schmidt (1995) GMM estimator (AS-GMM), the two-step Blundell and Bond (1998) GMM estimator (BB-GMM), and the Hsiao et al. (2002) QML estimator. Reported are the average bias of the estimates, the root mean square error (RMSE), and the empirical size of the Wald statistics for the hypothesis $\beta = \beta_0$. 'rob-Size' refers to the Wald test employing robust standard errors considered in Theorem 2.

Table 9: Simulation results: interactive random effects, $N = 50$

	α						β					
	WG	BC	AB-GMM	AS-GMM	BB-GMM	QML	WG	BC	AB-GMM	AS-GMM	BB-GMM	QML
	$\alpha = 0.4$											
$T = 10$												
Bias	-0.035	-0.001	-0.052	0.017	0.123	-0.010	0.017	0.002	-0.009	0.005	-0.042	-0.023
RMSE	0.056	0.045	0.106	0.159	0.142	0.044	0.063	0.061	0.092	0.157	0.104	0.066
Size	0.484	0.323	0.365	0.336	0.581	0.345	0.070	0.322	0.093	0.121	0.074	0.086
rob-Size	0.240	0.080	n.a.	n.a.	n.a.	n.a.	0.114	0.080	n.a.	n.a.	n.a.	n.a.
$T = 25$												
Bias	-0.014	-0.001	-0.024	0.024	0.138	-0.006	0.009	0.000	0.012	0.014	-0.037	-0.012
RMSE	0.029	0.026	0.048	0.125	0.145	0.026	0.042	0.042	0.053	0.124	0.069	0.044
Size	0.417	0.332	0.388	0.419	0.915	0.350	0.104	0.332	0.102	0.214	0.104	0.113
rob-Size	0.100	0.058	n.a.	n.a.	n.a.	n.a.	0.084	0.058	n.a.	n.a.	n.a.	n.a.
$T = 50$												
Bias	-0.008	-0.002	-0.019	0.018	0.135	-0.004	0.006	0.001	0.017	0.020	-0.039	-0.005
RMSE	0.020	0.019	0.034	0.105	0.140	0.019	0.030	0.029	0.038	0.103	0.057	0.030
Size	0.414	0.367	0.446	0.381	0.960	0.375	0.106	0.367	0.116	0.223	0.142	0.113
rob-Size	0.082	0.060	n.a.	n.a.	n.a.	n.a.	0.068	0.060	n.a.	n.a.	n.a.	n.a.
	$\alpha = 0.9$											
$T = 10$												
Bias	-0.249	-0.063	-0.470	-0.205	-0.034	-0.070	-0.018	-0.004	-0.053	-0.017	0.007	-0.058
RMSE	0.277	0.150	0.554	0.362	0.093	0.165	0.062	0.059	0.099	0.136	0.085	0.086
Size	0.943	0.431	0.882	0.625	0.278	0.700	0.080	0.401	0.206	0.103	0.045	0.241
rob-Size	0.666	0.056	n.a.	n.a.	n.a.	n.a.	0.145	0.059	n.a.	n.a.	n.a.	n.a.
$T = 25$												
Bias	-0.098	-0.016	-0.268	-0.137	-0.006	-0.036	0.003	0.002	-0.005	0.008	0.015	-0.037
RMSE	0.112	0.066	0.309	0.285	0.043	0.066	0.037	0.036	0.056	0.103	0.057	0.058
Size	0.922	0.455	0.924	0.726	0.256	0.664	0.074	0.411	0.121	0.179	0.083	0.204
rob-Size	0.478	0.022	n.a.	n.a.	n.a.	n.a.	0.091	0.029	n.a.	n.a.	n.a.	n.a.
$T = 50$												
Bias	-0.045	-0.005	-0.162	-0.071	0.005	-0.018	0.003	0.000	0.018	0.018	0.010	-0.027
RMSE	0.058	0.042	0.189	0.242	0.033	0.040	0.025	0.025	0.042	0.094	0.040	0.040
Size	0.832	0.591	0.908	0.718	0.214	0.699	0.064	0.513	0.138	0.254	0.075	0.246
rob-Size	0.279	0.025	n.a.	n.a.	n.a.	n.a.	0.075	0.034	n.a.	n.a.	n.a.	n.a.

Note: The comparison includes the within-groups estimator (WG), the bias-corrected method of moments estimator (BC), the one-step Arellano and Bond (1991) GMM estimator (AB-GMM), the two-step Ahn and Schmidt (1995) GMM estimator (AS-GMM), the two-step Blundell and Bond (1998) GMM estimator (BB-GMM), and the Hsiao et al. (2002) QML estimator. Reported are the average bias of the estimates, the root mean square error (RMSE), and the empirical size of the Wald statistics for the hypothesis $\beta = \beta_0$. 'rob-Size' refers to the Wald test employing robust standard errors considered in Theorem 2.

Table 10: Simulation results: interactive random effects, $N = 200$

	α						β					
	WG	BC	AB-GMM	AS-GMM	BB-GMM	QML	WG	BC	AB-GMM	AS-GMM	BB-GMM	QML
						$\alpha = 0.4$						
$T = 10$												
Bias	-0.035	-0.002	-0.044	-0.018	0.027	-0.010	0.014	0.000	-0.009	-0.041	-0.052	-0.025
RMSE	0.053	0.040	0.095	0.133	0.053	0.039	0.036	0.033	0.066	0.089	0.073	0.044
Size	0.754	0.562	0.561	0.533	0.250	0.583	0.105	0.562	0.231	0.201	0.175	0.166
rob-Size	0.278	0.064	n.a.	n.a.	n.a.	n.a.	0.135	0.064	n.a.	n.a.	n.a.	n.a.
$T = 25$												
Bias	-0.014	-0.001	-0.021	-0.010	0.041	-0.006	0.010	0.001	0.010	-0.008	-0.026	-0.011
RMSE	0.027	0.024	0.043	0.116	0.050	0.024	0.027	0.025	0.033	0.064	0.040	0.028
Size	0.660	0.577	0.620	0.617	0.646	0.603	0.200	0.577	0.186	0.211	0.139	0.200
rob-Size	0.124	0.058	n.a.	n.a.	n.a.	n.a.	0.097	0.058	n.a.	n.a.	n.a.	n.a.
$T = 50$												
Bias	-0.007	0.000	-0.015	-0.014	0.052	-0.003	0.006	0.000	0.015	0.018	-0.010	-0.006
RMSE	0.018	0.017	0.029	0.120	0.058	0.017	0.020	0.019	0.026	0.079	0.024	0.020
Size	0.656	0.605	0.641	0.673	0.890	0.610	0.203	0.605	0.239	0.461	0.128	0.211
rob-Size	0.077	0.051	n.a.	n.a.	n.a.	n.a.	0.073	0.051	n.a.	n.a.	n.a.	n.a.
						$\alpha = 0.9$						
$T = 10$												
Bias	-0.246	-0.059	-0.463	-0.266	-0.087	-0.071	-0.018	-0.004	-0.052	-0.029	-0.004	-0.058
RMSE	0.271	0.143	0.546	0.429	0.139	0.155	0.035	0.031	0.078	0.078	0.045	0.071
Size	0.976	0.565	0.951	0.866	0.623	0.843	0.105	0.540	0.445	0.115	0.020	0.516
rob-Size	0.676	0.050	n.a.	n.a.	n.a.	n.a.	0.180	0.050	n.a.	n.a.	n.a.	n.a.
$T = 25$												
Bias	-0.097	-0.014	-0.256	-0.155	-0.043	-0.035	0.002	0.001	-0.005	0.002	0.005	-0.039
RMSE	0.112	0.067	0.300	0.328	0.077	0.066	0.018	0.018	0.036	0.058	0.029	0.049
Size	0.957	0.634	0.961	0.879	0.728	0.842	0.050	0.566	0.216	0.176	0.042	0.536
rob-Size	0.456	0.027	n.a.	n.a.	n.a.	n.a.	0.081	0.033	n.a.	n.a.	n.a.	n.a.
$T = 50$												
Bias	-0.046	-0.006	-0.157	-0.094	-0.023	-0.019	0.004	0.000	0.018	0.019	0.008	-0.026
RMSE	0.058	0.041	0.183	0.265	0.052	0.039	0.013	0.013	0.029	0.056	0.024	0.033
Size	0.928	0.770	0.964	0.909	0.785	0.843	0.067	0.634	0.296	0.389	0.146	0.515
rob-Size	0.301	0.025	n.a.	n.a.	n.a.	n.a.	0.074	0.033	n.a.	n.a.	n.a.	n.a.

Note: The comparison includes the within-groups estimator (WG), the bias-corrected method of moments estimator (BC), the one-step Arellano and Bond (1991) GMM estimator (AB-GMM), the two-step Ahn and Schmidt (1995) GMM estimator (AS-GMM), the two-step Blundell and Bond (1998) GMM estimator (BB-GMM), and the Hsiao et al. (2002) QML estimator. Reported are the average bias of the estimates, the root mean square error (RMSE), and the empirical size of the Wald statistics for the hypothesis $\beta = \beta_0$. 'rob-Size' refers to the Wald test employing robust standard errors considered in Theorem 2.

Table 11: Simulation results: nonstationary initialization, $N = 50$

	α						β					
	WG	BC	AB-GMM	AS-GMM	BB-GMM	QML	WG	BC	AB-GMM	AS-GMM	BB-GMM	QML
	$\alpha = 0.4$											
$T = 5$												
Bias	-0.041	0.000	-0.056	0.031	0.386	0.000	-0.001	-0.003	-0.041	0.009	0.071	0.009
RMSE	0.048	0.027	0.175	0.101	0.388	0.027	0.098	0.099	0.172	0.156	0.187	0.099
$T = 10$												
Bias	-0.023	0.001	-0.037	0.087	0.364	0.003	0.010	0.000	-0.018	0.043	0.003	0.010
RMSE	0.029	0.018	0.083	0.157	0.366	0.019	0.062	0.062	0.090	0.138	0.118	0.062
$T = 25$												
Bias	-0.011	0.000	-0.009	0.084	0.346	0.001	0.007	-0.001	0.002	0.036	-0.026	0.003
RMSE	0.016	0.012	0.026	0.107	0.347	0.012	0.038	0.037	0.045	0.090	0.070	0.037
$T = 50$												
Bias	-0.006	0.000	-0.006	0.073	0.294	0.001	0.005	0.000	0.005	0.017	-0.040	0.002
RMSE	0.010	0.008	0.015	0.083	0.297	0.008	0.025	0.025	0.031	0.061	0.061	0.025
	$\alpha = 0.9$											
$T = 5$												
Bias	-0.328	0.011	-0.448	-0.013	0.114	0.032	-0.016	-0.002	-0.064	-0.012	-0.001	0.028
RMSE	0.334	0.119	0.621	0.149	0.124	0.145	0.098	0.102	0.132	0.129	0.136	0.100
$T = 10$												
Bias	-0.152	0.003	-0.308	-0.004	0.089	0.029	-0.001	0.000	-0.047	0.003	0.008	0.032
RMSE	0.156	0.052	0.379	0.094	0.092	0.073	0.062	0.061	0.091	0.089	0.088	0.066
$T = 25$												
Bias	-0.058	0.000	-0.265	0.006	0.055	0.011	0.002	-0.001	-0.036	0.009	0.011	0.020
RMSE	0.060	0.017	0.299	0.059	0.063	0.025	0.036	0.036	0.059	0.057	0.052	0.040
$T = 50$												
Bias	-0.030	0.000	-0.094	0.004	0.041	0.005	0.003	0.000	-0.003	0.007	0.006	0.013
RMSE	0.031	0.010	0.108	0.043	0.050	0.012	0.024	0.024	0.034	0.040	0.036	0.027

Note: The estimators in the comparison are the within-groups estimator (WG), our bias-corrected estimator (BC), the one-step Arellano and Bond (1991) GMM estimator (AB-GMM), the two-step Ahn and Schmidt (1995) GMM estimator (AS-GMM), the two-step Blundell and Bond (1998) GMM estimator (BB-GMM), and the Hsiao et al. (2002) QML estimator. Reported are the average bias of the estimates and the root mean square error (RMSE).

Table 12: Simulation results: nonstationary initialization, $N = 200$

	α						β					
	WG	BC	AB-GMM	AS-GMM	BB-GMM	QML	WG	BC	AB-GMM	AS-GMM	BB-GMM	QML
	$\alpha = 0.4$											
$T = 5$												
Bias	-0.041	0.000	-0.014	0.003	0.379	0.000	0.000	-0.002	-0.011	-0.004	0.101	0.010
RMSE	0.043	0.014	0.088	0.028	0.380	0.014	0.048	0.048	0.086	0.064	0.135	0.049
$T = 10$												
Bias	-0.023	0.000	-0.010	0.006	0.348	0.002	0.013	0.003	-0.003	-0.001	0.019	0.012
RMSE	0.025	0.009	0.041	0.034	0.348	0.010	0.033	0.030	0.045	0.047	0.065	0.032
$T = 25$												
Bias	-0.011	0.000	-0.003	0.012	0.346	0.001	0.009	0.001	0.001	0.000	-0.010	0.004
RMSE	0.012	0.006	0.012	0.018	0.346	0.006	0.020	0.018	0.024	0.029	0.037	0.018
$T = 50$												
Bias	-0.006	0.000	-0.001	0.024	0.360	0.001	0.005	0.000	0.001	0.007	-0.010	0.002
RMSE	0.007	0.004	0.007	0.027	0.360	0.004	0.014	0.013	0.016	0.025	0.028	0.013
	$\alpha = 0.9$											
$T = 5$												
Bias	-0.321	0.011	-0.210	0.020	0.115	0.022	-0.015	-0.001	-0.029	-0.001	0.009	0.028
RMSE	0.323	0.069	0.369	0.099	0.117	0.077	0.049	0.047	0.074	0.062	0.063	0.053
$T = 10$												
Bias	-0.149	0.000	-0.130	0.014	0.091	0.020	0.001	0.003	-0.018	0.001	0.008	0.035
RMSE	0.150	0.024	0.187	0.066	0.092	0.036	0.030	0.030	0.048	0.044	0.044	0.045
$T = 25$												
Bias	-0.056	0.000	-0.139	0.013	0.070	0.010	0.004	0.001	-0.020	0.002	0.009	0.022
RMSE	0.057	0.008	0.164	0.041	0.070	0.014	0.018	0.017	0.034	0.029	0.030	0.028
$T = 50$												
Bias	-0.030	0.000	-0.027	0.015	0.065	0.005	0.003	0.000	-0.002	0.003	0.010	0.012
RMSE	0.030	0.005	0.036	0.026	0.065	0.007	0.013	0.012	0.017	0.019	0.021	0.018

Note: The estimators in the comparison are the within-groups estimator (WG), our bias-corrected estimator (BC), the one-step Arellano and Bond (1991) GMM estimator (AB-GMM), the two-step Ahn and Schmidt (1995) GMM estimator (AS-GMM), the two-step Blundell and Bond (1998) GMM estimator (BB-GMM), and the Hsiao et al. (2002) QML estimator. Reported are the average bias of the estimates and the root mean square error (RMSE).