Advanced Dynamic Panel Data Methods

A Tentative Methodological Strategy for Panel Data Analysis

Sebastian Kripfganz

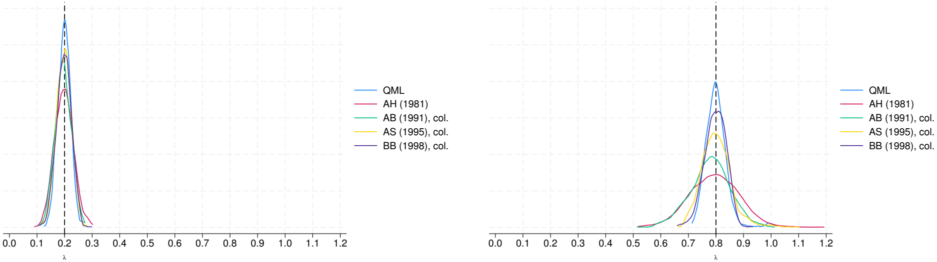
University of Exeter Business School, Department of Economics, Exeter, UK
www.kripfganz.de @Kripfganz

Universidad de Salamanca

July 17–19, 2023

Estimator Comparison

- Simulated distributions (kernel density estimates) based on 1,001 replications:
  - $y_{it} = \lambda y_{i,t-1} + \alpha_i + \varepsilon_{it}$, where $\varepsilon_{it} \sim \mathcal{N}(0,1)$, and $\alpha_i \in \{-1,0,1\}$; $N = 300$, $T = 10$
  - Stationary initial observations: $y_{i1} = \frac{\alpha_i}{1-\lambda} + \nu_{i1}$, where $\nu_{i1} \sim \mathcal{N}\left(0, \frac{1}{1-\lambda^2}\right)$
  - All GMM estimators are two-step estimators.

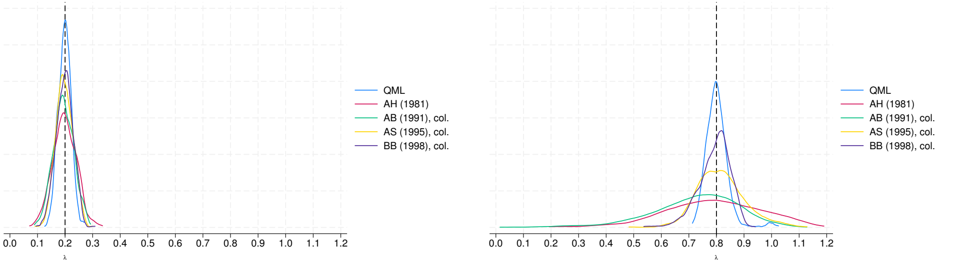


(a) $\lambda = 0.2$ (low persistence)

(b) $\lambda = 0.8$ (high persistence)

Estimator Comparison

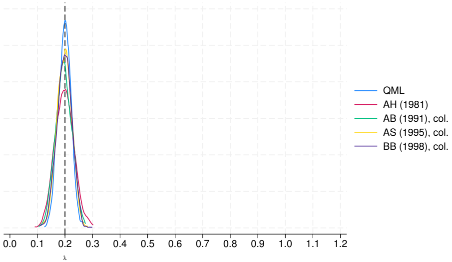
- Simulated distributions (kernel density estimates) based on 1,001 replications:
  - $y_{it} = \lambda y_{i,t-1} + \alpha_i + \varepsilon_{it}$, where $\varepsilon_{it} \sim \mathcal{N}(0,1)$, and $\alpha_i \in \{-2, 0, 2\}$; $N = 300$, $T = 10$
  - Stationary initial observations: $y_{i1} = \frac{\alpha_i}{1-\lambda} + \nu_{i1}$, where $\nu_{i1} \sim \mathcal{N}\left(0, \frac{1}{1-\lambda^2}\right)$
  - All GMM estimators are two-step estimators.



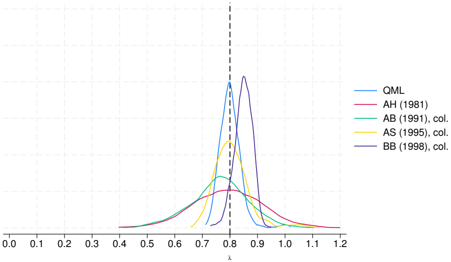
(a) $\lambda = 0.2$ (low persistence) (b) $\lambda = 0.8$ (high persistence)

Estimator Comparison

- Simulated distributions (kernel density estimates) based on 1,001 replications:
  - $y_{it} = \lambda y_{i,t-1} + \alpha_i + \varepsilon_{it}$, where $\varepsilon_{it} \sim \mathcal{N}(0,1)$, and $\alpha_i \in \{-1,0,1\}$; $N = 300$, $T = 10$
  - Nonstationary initial observations: $y_{i1} = 0$
  - All GMM estimators are two-step estimators.



(a) $\lambda = 0.2$ (low persistence)



(b) $\lambda = 0.8$ (high persistence)

Estimator Comparison

- In simple panel autoregressive models (or models with strictly exogenous regressors), the QML (or BC-MM) estimator outperforms GMM estimators with low bias and variance.
  - If T is very small, convergence problems might arise.
- GMM estimators tend to become less reliable when $\lambda \to 1$. The Ahn and Schmidt (1995) estimator with nonlinear moment functions and the Blundell and Bond (1998) "system GMM" estimator under a stronger initial-observations assumption can mitigate this problem.
  - A large variance ratio $\frac{\sigma_\alpha^2}{\sigma_\varepsilon^2}$ negatively affects estimators relying on instruments for the level model or level instruments for a transformed model.
  - Validity of the initial-observations assumption is crucial for the "system GMM" estimator, especially in the situation where it promises the most benefits; i.e., when $\lambda \to 1$.

Model and Moment Selection

- Unless (economic) theory gives a clear prescription of the model to be estimated, a specification search might be necessary as part of the empirical analysis (Kiviet, 2020).
  - Higher-order lags of the dependent variable, $y_{i,t-2}, y_{i,t-3}, \ldots$, and the other regressors, $\mathbf{x}_{i,t-1}, \mathbf{x}_{i,t-2}, \ldots$, might have predictive power and could help to prevent serial correlation of the error term ε_{it} when included as regressors.

$$y_{it} = \sum_{j=1}^{p} \lambda_j y_{i,t-j} + \sum_{j=0}^{q} \mathbf{x}'_{i,t-j} \boldsymbol{\beta}_j + \alpha_i + \varepsilon_{it}$$

  - Time dummies should be included by default unless there is sufficient evidence against them.
  - Interaction terms among the explanatory variables (possibly including time dummies) might be necessary to allow for heterogeneity in the dynamic impact multipliers.
  - The regressors $\mathbf{x}_{it}$ need to be classified correctly as strictly exogenous, predetermined, or endogenous.

Model and Moment Selection

- Omitted variables (such as higher-order lags of already included variables as well as other excluded variables) can cause correlation of the instruments with the error term, and thus turn the estimator inconsistent. However, if they are irrelevant, including them reduces the estimator's efficiency.
- Stronger assumptions yield more instruments – more precisely, overidentifying restrictions – which (asymptotically) improve the estimator's efficiency. However, if they are violated, they turn the estimator inconsistent.

Model and Moment Selection

- The Andrews and Lu (2001) model and moment selection criteria (MMSC) can support the specification search for GMM estimation. These criteria subtract a bonus term from the overidentification test statistic J that rewards fewer coefficients $p + K_x(1+q)$ for a given number of moment conditions K_z, or more overidentifying restrictions for a given number of coefficients.
  - These MMSC resemble the idea of the traditional Akaike, Bayesian, or Hannah-Quinn information criteria:

$$\begin{aligned}
\text{MMSC-AIC} &= J - 2(K_z - p - K_x(1+q)) \\
\text{MMSC-BIC} &= J - (K_z - p - K_x(1+q)) \ln N \\
\text{MMSC-HQIC} &= J - 2Q(K_z - p - K_x(1+q)) \ln \ln N
\end{aligned}$$

    for some $Q > 1$
  - Importantly, the MMSC help to select relevant regressors and valid moment conditions. They are not designed to compare different instrument reduction techniques.

Model and Moment Selection

- During the model selection process, as many decisions as possible should be justified (ex ante) on theoretical grounds or guided by statistical evidence.
  - The fewer "researcher degrees of freedom" are used, the harder it is to attack the specification during a scientific review process.
  - Transparency is key for reliability and replicability.
- Especially the choice of any instrument reduction technique is highly arbitrary.
  - Ideally, this choice should not matter in a substantial way, because all fundamental assumptions are left unchanged.
  - However, especially with relatively small N, GMM estimators tend to be sensitive to this choice. This creates the risk that researchers choose the "most favorable" specification (ex post).
  - As much as possible, ex-ante commitment to a specific instrument reduction approach is desirable. In particular, use the same approach for all variables to avoid accusations of "specification mining".

Sequential Model Specification

- The following sequential specification process is adapted from Kiviet (2020), with some modifications.
  - Each application has its idiosyncrasies. Some of the following steps might be irrelevant, additional steps might be needed, or a different order of the steps might be appropriate.

1. Specify an initial candidate "maintained statistical model" (MSM).
   - An initial candidate MSM should avoid the omission of relevant regressors – including lags, potentially relevant interaction term, and time dummies – and treat variables $\mathbf{x}_{it}$ as endogenous (unless there is opposing theory or evidence), but it should also avoid an overparametrization.
   - Keep in mind that increasing the lag orders p and/or q reduces the sample size which can be costly when T is small.
   - Especially when N is small or T is relatively large, collapse and/or curtail the instruments. Initially, err on the side of caution – i.e., use rather few overidentifying restrictions.

Sequential Model Specification

2. Compute the two-step, iterated, or continuously-updating GMM estimator (with appropriately corrected standard errors) for the initial candidate MSM, and check whether it passes the specification tests.
   - If there are concerns about an imprecisely estimated optimal weighting matrix, the one-step GMM estimator with robust standard errors might be used instead, especially when N is relatively small.
   - Check the serial correlation tests at least up to order 2.
   - Check the overidentification and underidentification tests.
   - If any of the tests is not satisfied, go back to step 1 and amend the initial candidate MSM.
   - p-values for the specification tests should be chosen conservatively (to guard against accidental misspecification and against concerns for statistical inference arising from multiple testing); see Kiviet (2020) for a discussion.

Sequential Model Specification

3. Remove lags or interaction effects with (very) high p-values in individual or joint significance tests, and/or check whether further lags or interaction effects improve the model fit, adjusted for the degrees of freedom.
   - Reduce the model sequentially, i.e. remove the longest lag or interaction effect with the highest p-value first and reestimate the model. Repeat the procedure until none of the longest lags has (very) high p-values any more.
   - For every new candidate model, carry out the specification tests as in step 2.
   - Use the MMSC to compare the candidate models that pass the specification tests.
   - Check whether the results for the preferred model are robust to alternative (less restrictive) ways of instrument reduction.

Sequential Model Specification

4. Separately for all regressors classified as endogenous, add the extra instruments that become valid if the regressors were predetermined (unless theory clearly indicates that a variable should be endogenous), and check the corresponding incremental overidentification tests.
   - Keep an eye on other specification tests and MMSC as well.
   - Treat the variable with the highest acceptable *p*-value of the incremental overidentification tests as predetermined, and repeat the procedure for the remaining variables until no more variable can be confidently classified as predetermined.
5. Separately for all regressors classified as predetermined, add the extra instruments that become valid if the regressors were strictly exogenous, and follow the procedure of step 4.
   - Have a look at underidentification tests as well. Passing the underidentification tests might require stronger exogeneity assumptions, possibly creating a conflict with overidentification tests.

Sequential Model Specification

6. Possibly, repeat step 3 based on the new MSM from step 5.
   - If predicted by theory, it might be worth exploring other coefficient restrictions besides those of equality to zero.
   - Keep in mind that statistical insignificance per se is not a sufficient reason to exclude a variable, in particular if the point estimate is (economically) large or if the effect of this variable is of particular interest in the analysis.
   - If it is difficult to establish a reliable MSM due to suspected instrument weakness, the nonlinear Ahn and Schmidt (1995) moment conditions could be added to gain identification strength.

Sequential Model Specification

8. Unless there is opposing theory or evidence, consider the additional instruments that are valid under the Blundell and Bond (1998) initial-observations assumption.

   - The additional instruments should only be added once a reliable MSM based has been established without the extra assumption.
   - Use incremental overidentification tests to separately investigate the additional instruments $\Delta \mathbf{x}_{it}$ (or $\Delta \mathbf{x}_{i,t-1}$) one by one for the level model first. Only if there is sufficiently strong evidence that all of those instruments are valid, add the extra instruments $\Delta y_{i,t-1}$.
   - Keep an eye on the other specification tests as well.

Sequential Model Specification

9. If (many of) the additional level moment conditions in step 8 are rejected, add instead the nonlinear Ahn and Schmidt (1995) moment conditions if supported by serial-correlation tests.
    - A rejection of this model by the specification tests causes doubt on the MSM and might require to revoke some of the decisions made in earlier steps.
    - To further improve the efficiency, it might be worth utilizing the nonlinear Ahn and Schmidt (1995) moment conditions valid under homoskedasticity, but keep in mind that it is often difficult to justify such a strong assumption.
    - It might be reasonable to add the nonlinear moment conditions already at a previous step to circumvent identification problems.

Sequential Model Specification

10. Review the decisions made at earlier steps and check the robustness to alternative reasonable choices.
    - If different decisions lead to different outcomes of the specification process, use MMSC to find a final specification and/or report the results from multiple specifications. The latter is helpful to provide an idea about the model uncertainty.
    - The process is not designed to recover a true, structural model. "Structure" comes from any theoretical (ex ante) constraints on the model specification process.
    - If no robust MSM seems attainable, it might help to reduce T and/or N in order to obtain a simple in which it is easier to establish coefficient homogeneity.

Sequential Model Specification

11. If there are any time-invariant regressors of particular interest (beyond the mere desire to control for them), add them and sufficiently many instruments for the level model.
    - Keep in mind that the inclusion of time-invariant regressors generally requires potentially strong identifying assumption.
    - If the coefficients of the time-invariant regressors are overidentified, check the incremental overidentification tests (and possibly underidentification tests as well).
12. If all regressors $\mathbf{x}_{it}$ are justifiably classified as strictly exogenous, consider the BC-MM or QML estimators as more efficient alternatives to GMM.
    - These estimators can serve as a useful benchmark in any case.
    - However, note that their requirements on the error term are stonger. In particular, they usually require time-series homoskedasticity in addition to absence of serial correlation.

Interim Conclusion

- The sequential model selection approach creates transparency and helps to reduce the arbitrariness of some model specification choices.
- The process is not without shortcomings. At most steps, multiple specification tests are involved. As with classical hypothesis testing, each test is subject to type-I and type-II errors. These errors multiply the more tests we carry out.
  - Statistical inference on the final model's coefficients is conditional on the selected model, ignoring model uncertainty. It is therefore important to be transparent about the robustness (or lack thereof) of the model selection process.
  - The problems of statistical testing are aggravated (in finite samples) if the tests suffer from poor size control and power.
- Just because a test does not reject an assumption, this does not imply that it is true.
  - (Strong) theoretical arguments for or against an assumption should take precedence.